Counting

- Lemma: If T is a binary tree then at level k, T has \( \leq 2^k \) nodes.

- Theorem: If T has height h, then number of nodes in T \( \leq 2^{h+1} - 1 \).
- Equivalently, if T has n nodes then 
  \[ n - 1 \geq h \geq \log_2(n+1) - 1 \]

More Terminology

- Full binary tree of height h has all leaves on level h.
- Complete binary tree of height h is obtained from a full binary tree of height h with 0 or more (but not all) of the rightmost leaves at level h removed.
- T is balanced if it has the minimum possible height for its # of nodes.
- In this case, height = ceiling of \( \log_2(n+1) - 1 \) or \( O(\log_2 n) \)

Binary Trees in Java

- No implementation in standard Java libraries
- Structure5 has BinaryTree<E> class, but no interface.
- Like doubly-linked list:
  - value: E
  - parent, left, right: BinaryTree<E>
Tree Traversals

- Traversals:
  - Pre-Order: root, left subtree, right subtree
  - In-Order: left subtree, root, right subtree
  - Post-Order: left subtree, right subtree, root

- Most algorithms have two parts:
  - Build tree
  - Traverse tree, performing operations on nodes

Evaluate Expression Tree

- Evaluate left subtree, right subtree, perform operation at root.
- Generate stack-based code to evaluate: post-order
Animals Game

- Guess animal using only true-false questions.
- See demo program