(1) Describe the set of all points \((x, y, z)\) in \(\mathbb{R}^3\) that satisfy the following equations.

(a) \(x = 0\)  
(b) \(y^2 + z^2 - 4z = 5\)  
(c) \(x^2 + y^2 + z^2 - 4z = 5\)  
(d) \(xy = 0\)  
(e) \(xyz = 0\)  
(f) \(x^2 - z^2 = 0\)

(2) Consider the two points \(A = (x_1, y_1, z_1)\) and \(B = (x_2, y_2, z_2)\).

(a) Verify that \(M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)\) is equidistant from \(A\) and \(B\).

(b) Show that \(M\) is in fact the midpoint of the segment \(AB\) (hint: use the triangle inequality).

(3) Consider the three points \(A = (2, 2, 3)\), \(B = (1, 0, 1)\), and \(C = (x, y, z)\). Write down equations involving \(x\), \(y\), and \(z\) that ensure that the triangle \(ABC\) is isosceles with base \(AB\). Describe the set of all such points \(C\) geometrically.

(4) Find an algebraic expression for the set of all points \((x, y, z)\) whose distance from \((1, 2, 3)\) is twice their distance from \((1, 0, -1)\). Describe that set geometrically.

(5) Match each function with its graph, justifying your choices.

| (I) \(z = 1/(1 + x^2 + y^2)\) | (II) \(z = \sin(x) + \sin(y)\) | (III) \(z = (x^2 - y^2)^2\) | (IV) \(z = (x - y)^2\) | (V) \(z = |x| + |y|\) | (VI) \(z = |xy|\) |
|-------------------------|----------------|----------------|----------------|----------------|----------------|

(6) Through the course of the homework assignments, and eventually, the class, we will get to know certain formal objects called forms. We begin that journey here! In this problem, we introduce a new language whose purpose is, it would seem, to unnecessarily complicate matters.

We define forms over \(\mathbb{R}\) as follows. A 0-form is another name for a nice function \(f(x) : \mathbb{R} \to \mathbb{R}\), and a 1-form is an object like \(g(x)dx\), where \(g(x)\) is a nice function.

For instance, \(\cos(x)\) and \(x^3 + x\) are 0-forms, and \(-\sin(x)dx\) and \((3x^2 + 1)dx\) are 1-forms.

(a) Convince yourself (and your reader) that for \(k = 0, 1\), you can make new \(k\)-forms from old ones by adding two \(k\)-forms together or multiplying \(k\)-forms by scalars. For instance \(\cos(x) + x^3 + x\) is a 0-form, and \(47\sin(x)dx\) is a 1-form.

\(^1\)“Nice” is our catchall, imprecise adjective for a function that has the properties we need it to have—continuous, differentiable, whatever—for the results to follow to follow.
(b) If I give you a nice 0-form $f(x)$, you can make a 1-form by differentiating $f(x)$. More precisely, the 1-form $f'(x)dx$ can be obtained from the 0-form $f(x)$. To streamline (or complicate) the notation, we denote $df = f'(x)dx$. (Note that we write $df$ instead of $d(f)$, even though we think of $df$ as the output of the function $d$ acting on the 0-form $f$.) In other words, we think of $d$ as a function from the set of 0-forms to the set of 1-forms.

$$d : \{\text{0-forms}\} \longrightarrow \{\text{1-forms}\}$$

For instance, $d \cos(x) = -\sin(x)dx$, and $d(x^3 + x) = (3x^2 + 1)dx$. Does $d$ preserve addition? Does it preserve scalar multiplication?

(c) Are we using confusing notation by using $d$ to denote both the function from 0-forms to 1-forms and part of the 1-form $f(x)dx$? For instance, is there ambiguity if we say $dx$? Is that the function $d$ applied to the 0-form $f(x) = x$, or is that the 1-form $g(x)dx$ where $g(x) = 1$? Or are these two objects the same?

(d) Here’s a secret about forms: they have an affinity for being evaluated over regions of the same dimension. What I mean is the following. We evaluate the 0-form $f(x)$ over the boundary of the interval $[a, b]$, namely, over the (0-dimensional points) $a$ and $b$, by computing $f(b) - f(a)$. And we evaluate the 1-form $g(x)dx$ over the (1-dimensional) interval $[a, b]$ by computing the integral $\int_{x=a}^{x=b} g(x)dx$.

What assertions can you make about these two evaluations if the 1-form $g(x)dx$ was obtained from the 0-form $f(x)$ by way of the function $d$? Hint: This is an overcomplicated way of asking you to state the Fundamental Theorem of Calculus using the language of forms.