

$$e ::= x \mid e_1 e_2 \mid \lambda x. e$$

fv : LC → 2^V

$$fv(x) = \{x\}$$

$$fv(e_1 e_2) = fv(e_1) \cup fv(e_2)$$

$$fv(\lambda x. e) = fv(e) \setminus \{x\}$$

subst : LC → V → LC → LC

$$\text{subst}(x, y, e') = e' \quad \text{when } x=y$$

$$\text{subst}(x, y, e') = x \quad \text{when } x \neq y$$

$$\text{subst}(e_1 e_2, y, e') = \text{subst}(e_1, y, e') \text{ subst}(e_2, y, e')$$

$$\text{subst}(\lambda x. e, y, e') = \lambda x. e \quad \text{when } x=y$$

$$\text{subst}(\lambda x. e, y, e') = \lambda x. \text{subst}(e, y, e') \quad \text{when } x \neq y$$

$$e_1[e_2/x] \triangleq \text{subst}(e_1, x, e_2)$$

$$\lambda x. e =_a \lambda y. e[y/x] \quad \text{when } y \notin e$$

$$(\lambda x. e_1) e_2 =_{\beta} e_1[e_2/x]$$