Lecture 1: Propositional Logic

CSCI 55
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Syllabus at http://www.cs.pomona.edu/classes/cs055/

Logic

- "... the systematic study of the form of arguments."
- In particular the study of valid arguments.
  - How can you tell?
- Many logics of different expressiveness
  - Propositional logic
  - Predicate logic

Propositional Logic

- Letters p, q, r, etc. represent propositions
  - Today is Wednesday.
  - This class is horrible.
  - If I stay in this class, then I'll learn a lot.
  - ~~Where is Michael Greenberg today?~~
  - ~~Do your homework regularly!~~

(De)Composing Propositions

- Build more complicated propositions from simpler ones.
  - Mary is here *and* the sky is blue.
  - Mary is *not* here.
  - Mary is here *or* her plane was delayed.
  - *If* the sky is blue *then* I will *not* carry an umbrella.

Represent Symbolically

- Let p, q represent propositions then use logical connectives $\neg$, $\wedge$, $\vee$, $\rightarrow$ as follows to build up new formulas as follows:
  - $p \wedge q$ *and*
  - $\neg p$ *not*
  - $p \vee q$ *or (inclusive or)*
  - $p \rightarrow q$ *implies* or *if … then …*
  - $p \leftrightarrow q$ *iff*

Boolean Values

- Often convenient to use abbreviations for true and false:
  - true = $\top$
  - false = $\bot$
  - Can use in formulas: $p \rightarrow \bot$

English to Propositional Logic

- Let
  - p = Mary is here, q = The sky is blue,
    r = I will carry an umbrella, s = Mary's plane was delayed

- New propositions are:
  - $p \wedge q$
  - $\neg p$
  - $p \vee s$
  - $q \rightarrow (\neg r)$

- In Java:
  - `p && q`
  - `!p`
  - `p || s`
  - `!q || !r`

Disambiguating Propositional Logic

- Use parentheses to make clear how to apply logical connectives.
  - $q \rightarrow (\neg r)$
  - $\neg(p \wedge q)$
- Precedence of operators: $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$
  - So $\neg p \wedge q$ is really $(\neg p) \wedge q$
  - $p \wedge q \rightarrow \neg r$ is $(p \wedge q) \rightarrow (\neg r)$
  - Better to parenthesize if any confusion

Translation Issues

- $p \rightarrow q$ can be expressed as
  - "if p then q"
  - "if p, q"
  - "q, if p"
  - "p implies q"
  - "p, only if q"
  - "q, when p"
  - q is a necessary condition for p
  - p is a sufficient condition for q
  - q follows from p
- $p \leftrightarrow q$ can be expressed as
  - p iff q
  - p is a necessary and sufficient condition for q

More Complicated

- Try these:
  - p unless q *The student will fail unless he studies*
  - if p then q, and conversely

Semantics via Truth Tables

- Propositions can be true or false.
- Analyze according using “compositional” semantics
  - Meaning of whole follows from meaning of parts.

Meaning of $p \wedge q$

- Depends on meaning of each of p and q.
  - If p, q both true then $p \wedge q$ is true, otherwise false.
    - Doesn't depend on content of p, q, just truth value
  - What about $p \vee q$ and $\neg p$?
  - What about $p \leftrightarrow q$?
  - $p \rightarrow q$ harder ...

Truth Tables

P	Q	$P \wedge Q$
$\top$	$\top$	$\top$
$\top$	$\bot$	$\bot$
$\bot$	$\top$	$\bot$
$\bot$	$\bot$	$\bot$

One row for each combination of values of p and q

Truth Tables

P	Q	$P \vee Q$
$\top$	$\top$	$\top$
$\top$	$\bot$	$\top$
$\bot$	$\top$	$\top$
$\bot$	$\bot$	$\bot$

Truth Tables

P	$\neg P$
$\top$	$\bot$
$\bot$	$\top$

Why only two rows?
How many rows if n proposition letters?

Truth Tables

P	Q	$P \rightarrow Q$
$\top$	$\top$	$\top$
$\top$	$\bot$	$\bot$
$\bot$	$\top$	$\top$
$\bot$	$\bot$	$\top$

Material implication: No notion of causality.
Only worry about when it fails.

Truth Tables

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \rightarrow Q$	$P \leftrightarrow Q$
$\top$	$\top$	$\bot$	$\top$	$\top$	$\top$	$\top$
$\top$	$\bot$	$\bot$	$\bot$	$\top$	$\bot$	$\bot$
$\bot$	$\top$	$\top$	$\bot$	$\top$	$\top$	$\bot$
$\bot$	$\bot$	$\top$	$\bot$	$\bot$	$\top$	$\top$

Each row corresponds to different valuation

Classification of Formulas

- A formula ϕ is *valid*, or a *tautology*, if for all assignments to proposition letters, ϕ is true.
- A formula ϕ is *unsatisfiable*, or a *contradiction*, if for all assignments to proposition letters, ϕ is false.
- A formula ϕ is *contingent* if for some assignments to proposition letters ϕ is true, and others make it false.

Examples

- Tautologies:
  - $p \vee (\neg p)$
  - $p \to (q \to p)$
  - $\neg(p \wedge q) \leftrightarrow (\neg p \vee \neg q)$
  - $\neg(p \vee q) \leftrightarrow (\neg p \wedge \neg q)$
- Contradiction:
  - $p \wedge \neg p$

Logical Equivalence

- Two formulas ϕ and τ are *logically equivalent* iff for all assignments to proposition letters, ϕ and τ have the same truth values.
  - Write it as $\phi \equiv \tau$
  - How can we tell?
  - Equivalently ϕ and τ are logically equivalent iff $\phi \leftrightarrow \tau$ is a tautology.

Example

- Example $\neg p \rightarrow q \equiv p \vee q$

P	Q	$\neg P$	$\neg P \rightarrow Q$	$P \vee Q$
$\top$	$\top$	$\bot$	$\top$	$\top$
$\top$	$\bot$	$\bot$	$\top$	$\top$
$\bot$	$\top$	$\top$	$\top$	$\top$
$\bot$	$\bot$	$\top$	$\bot$	$\bot$

How would you show $(\neg p \rightarrow q) \leftrightarrow (p \vee q)$ is a tautology?

DeMorgan's Laws

- Equivalences using negation
  - $\neg(p \wedge q) \equiv (\neg p \vee \neg q)$
  - $\neg(p \vee q) \equiv (\neg p \wedge \neg q)$
  - $\neg(p \rightarrow q) \equiv (\neg q \rightarrow \neg p)$
- Others:
  - $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
  - $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
- See Table 6-8 in text for more

Logical Implication

- How could you define that?

LaTeX

- Text formatting system designed by Donald Knuth & added to by Leslie Lamport.
  - Not WYSIWYG!!
  - But lovely output.
- Need to learn for CS classes including 55, 81, 140, and for senior project/thesis.
  - Useful outside of CS as well

Benefits of LaTeX

- Takes care of tricky aspects of technical prose (if you don't fight it!).
    - Automatic numbering
    - Automatic handling of citations
    - Lots of macros for common formatting
        - You can write your own!

Good News!

- Mainly fill in template with your answers.
- Learn most of what you need from raw LaTeX of assignment sheet.
- See syllabus for details
  - http://www.cs.pomona.edu/classes/cs055/latex.html

Translation

- Iran will supply arms to Syria only if Syria helps Hezbollah.
- Only if Jenna passes the exam will Jenna get her license.
- Bill will take geology just in case it fulfills the science requirement.