

## CS054: Truth tables

The goal of this worksheet is to give you practice with truth tables: what are truth tables and how do you construct them? It's not for a grade—no need to turn it in! I'll post solutions, but you'll get the most out of it if you don't peek.

For all of these questions, I'll use programmatic notation—`&&` for `andb` in Coq, `||` for `orb` in Coq, and `!` for `negb`. We write `t` for `true` and `f` for `false`; you can also use `T` and `F` or  $\top$  and  $\perp$ . The questions are asking about the definitions we have in `Basics.v`.

I expect you to simply *know* the truth tables for `||`, `&&`, `!`, and `=>`.

1. **Sample:** Consider the boolean expression `!a || b` given booleans  $a$  and  $b$ .

- (a) How many rows will the truth table have? **Answer:** four, because there are two variables,  $a$  and  $b$ , and we must consider each value they consider.
- (b) How many columns will the truth table have and what are they? **Answer:** four; one each for  $a$  and  $b$ , one for `!a`, and one for the whole expression.
- (c) What is the truth table? **Answer:**

$a$	$b$	<code>!a</code>	<code>!a    b</code>
<code>t</code>	<code>t</code>	<code>f</code>	<code>t</code>
<code>t</code>	<code>f</code>	<code>f</code>	<code>f</code>
<code>f</code>	<code>t</code>	<code>t</code>	<code>t</code>
<code>f</code>	<code>f</code>	<code>t</code>	<code>t</code>

- (d) Can you rephrase this expression in terms of other boolean operators? That is, can you find a smaller boolean expression that has an equivalent truth table? **Answer:** The expression  $a \Rightarrow b$  (i.e., `impb a b`) has the same truth table.

2. Consider the boolean expression `p || (p && q)`.

- (a) How many rows will the truth table have?
- (b) How many columns will the truth table have?
- (c) What is the truth table?

- (d) Can you rephrase this expression in terms of other boolean operators? That is, can you find a smaller boolean expression that has an equivalent truth table?

3. Consider the boolean expression  $x \&\& (x \mid\mid y)$ .

- (a) How many rows will the truth table have?
- (b) How many columns will the truth table have?
- (c) What is the truth table?

(d) Can you rephrase this expression in terms of other boolean operators? That is, can you find a smaller boolean expression that has an equivalent truth table?

4. Consider the boolean expression  $(x \&\& y) \mid\mid (x \&\& z)$ .

- (a) How many rows will the truth table have?
- (b) How many columns will the truth table have?
- (c) What is the truth table?

(d) Can you rephrase this expression in terms of other boolean operators? That is, can you find a smaller boolean expression that has an equivalent truth table?

5. Consider the boolean expression  $!x \&\& !y$ .

- (a) How many rows will the truth table have?
- (b) How many columns will the truth table have?
- (c) What is the truth table?

(d) Can you rephrase this expression in terms of other boolean operators? That is, can you find a smaller boolean expression that has an equivalent truth table?

6. Consider the boolean expression  $!a || !b$ .

- (a) How many rows will the truth table have?
- (b) How many columns will the truth table have?
- (c) What is the truth table?

(d) Can you rephrase this expression in terms of other boolean operators? That is, can you find a smaller boolean expression that has an equivalent truth table?

7. Use a truth table to prove that  $\parallel$  is commutative, i.e.,  $p \parallel q$  is the same as  $q \parallel p$ .

8. Use a truth table to prove that  $\parallel$  is associative, i.e.,  $p \parallel (q \parallel r)$  is the same as  $(p \parallel q) \parallel r$ .

Other good practice exercises (for which no solutions will be provided):

- Use truth tables to prove that  $\&\&$  and  $\otimes$  (a/k/a  $\text{xorb}$ ) are commutative and associative.
- Use truth tables to prove that  $!$  is involutive, i.e.,  $!(! b)$  is equivalent to  $b$ .
- Is  $\Rightarrow$  (a/k/a  $\text{impb}$ ) commutative or associative? Use truth tables to prove or disprove it.