CS054 Graphs

1. Draw an undirected graph with six or more nodes where every node has a degree of at least 2.
2. The following questions all concern the number of possible edges in a variety of different types of graphs with n vertices. No graph will have multiple edges between nodes or hyperedges. Read closely!
   (a) What's the maximum number of edges in a directed graph with n vertices, if we disallow self-loops?
   (b) What's the maximum number of edges in a directed graph with n vertices, if we allow self-loops?
   (c) What's the maximum number of edges in an undirected graph with n vertices, if we disallow self loops?
   (d) What's the maximum number of edges in an undirected graph with n vertices, if we allow self loops?
3. Suppose G is a simple (i.e., undirected) bipartite graph with n_1 vertices in one partition and n_2 vertices in the other.
   (a) What's the maximum number of edges in G? Give a formula.
   (b) Whats the minimum number of edges in G? Give a formula.
4. Determine whether $K_{2,2}$ (the complete bipartite graph on two sets of two nodes each) is a subgraph of K_4 (the complete graph on four nodes).
5. Draw a strongly connected directed graph that has a simple circuit of length 4 but no circuits of length 3 or less.
6. Draw a tree with seven vertices. Draw the root at the top; there should be three leaves at the bottom.
7. For which numbers $n \geq 1$ is the complete graph K_n on n vertices a tree?
8. For which numbers $m, n \geq 1$ is the complete bipartite graph $K_{m,n}$ a tree? Explain (but don't prove) why. Don't write more than five lines of text.