Lambda Calculus Cheat Sheet

CSC 131

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1 Lambda calculus syntax

Lambda calculus terms are variables, function applications, or function definitions:

M ::= v | (M M) | λv. M

where "v" represents a variable symbol.

Computation takes place by substituting in actual parameters for *free occurrences* of formal parameters, which are defined by induction on the structure of lambda calculus terms as follows:

Definition 1.1 *If* M *is a term, then* FV(M)*, the collection of free variables of* M*, is defined as follows:*

1. FV(x) = { x }
2. FV(M N) = FV(M) $\cup$ FV(N)
3. FV(λv. M) = FV(M) - { *v* }

Definition 1.2 *We write* [N/x] M *to denote the result of replacing all free occurrences of identifier* x *by* N *in expression* M*.*

1. [N/x] x $\triangleq$ N,
2. [N/x] y $\triangleq$ y, *if* y $\neq$ x,
3. [N/x] (L M) $\triangleq$ ([N/x] L) ([N/x] M),
4. [N/x] (λy. M) $\triangleq$ λy. ([N/x] M), *if* y $\neq$ x *and* y $\notin$ FV(N),
5. [N/x] (λx. M) $\triangleq\lambda$x. M.

2 Rules of Computation

Definition 2.1 *The reduction rules for the lambda calculus are given by:*

(α) $\lambda \mathrm{x}.\, \mathrm{M} \xrightarrow{\alpha\!\!\rightarrow} \lambda \mathrm{y}.\, ([\mathrm{y}/\mathrm{x}]\, \mathrm{M})$, *if* $\mathrm{y} \notin \mathrm{FV}(\mathrm{M})$.

(β) $(\lambda \mathrm{x}.\, \mathrm{M})\ \mathrm{N} \xrightarrow{\beta\!\!\rightarrow} [\mathrm{N}/\mathrm{x}]\, \mathrm{M}$.

(η) $\lambda \mathrm{x}.\, (\mathrm{M}\ \mathrm{x}) \xrightarrow{\eta\!\!\rightarrow} M$.