Lecture 36: Epistemic Logic

CS 181O
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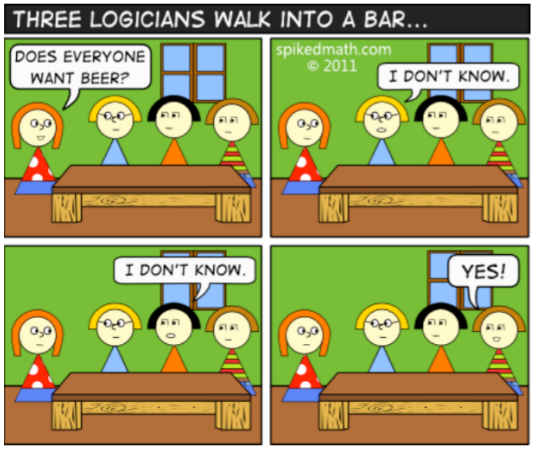
Some slides based on those of Christina Unger



Epistemic Dynamic Logic

- Let a be accessibility relation for an agent, ϕ is statement that can be true or false, while α is an accessibility relation.
- $\phi ::= \top, p, \neg\phi, \phi_1 \wedge \phi_2, [\alpha]\phi, [!\phi_1]\phi_2$
- $\alpha ::= a \mid ?\phi \mid \alpha_1; \alpha_2 \mid \alpha_1 \cup \alpha_2 \mid \alpha^*$
- Interpret in model $\mathcal{M}$ = <W,V,R> where W is possible worlds, V assigns values to prop letters, and R assigns each agent an equivalence relation to show accessible worlds

Common Knowledge

- $[!\phi]\psi$ true iff after public announcement of ϕ, ψ is true. *Restricts to worlds satisfying ϕ*
- Suppose have update: $[![(i \cup j)^*]\phi]\psi$
  - Announce it is common knowledge between i & j that ϕ.
  - If ϕ is already common knowledge then no impact
  - If not common knowledge then remove all worlds!
    - because restricts to all those worlds for which it is common knowledge.
    - Must be careful
- From now on write $C_{ij}\,\phi$ in place of $![(i \cup j)^*]\phi$

Common Knowledge & Presuppositions

- Presupposition is common knowledge between speaker and listener.
  - Jan washed her car
  - $\exists c.\ C_{ij}(car(c) \wedge owns(jan,c) \wedge female(jan)) \wedge washed(jan,c))$
  - First part common knowledge, last not!

General Situation

- If γ has ϕ as a presupposition, then interpret γ as $(C_{ij}\ \phi) \wedge \gamma$
- What happens if state presupposition first?
  - Jan has a car. Jan washed her car.
  - $![\exists c.(car(c) \wedge owns(jan,c))]\ \exists c.\ C_{ij}(car(c) \wedge owns(jan,c) \wedge female(jan)) \wedge washed(jan,c))$

Presupposition Accommodation

- Suppose presupposition p is common knowledge $(C_{ij}\ p)$. Then updating with $!([C_{ij}\ p] \wedge q)$ is equivalent to $!q$ *(no thinning)*
- If p not common knowledge then $!([C_{ij}\ p] \wedge q)$ leads to inconsistent state.
- On the other hand $!p$, followed by $!([C_{ij}\ p] \wedge q)$ is again consistent *(because !p made it common knowledge)*
- *Accommodate p by replacing by* $[!p][!([C_{ij}\ p] \wedge q)]$

Questions

- Answers to questions changes possible worlds.
  - What is difference between statement S and question "is S?" and responding.
  - [!S] restricts world to those where S is true
  - [is S?] changes focus of conversation to expecting an answer.
  - Response of [!yes] or [!no] restricts worlds to those where S is true or those where S is false.

Actually more Complex

- Did John go to the store?
  - He stayed home and watched a movie.
- Public announcement ends up restricting worlds to those where John not go to the store

Programming Common Knowledge

- See CAIA.hs
- See definition of m0
  - type: displayS5 m0
- and see effect of
  - displayS5 (upd_pa m0 (Disj [p, Neg q]))
    - *world 2 disappears*

Solving Muddy Children

- displayS5 initMuddy *(at end of file CAIA.hs)*
  - pi = the ith child is muddy
  - each child can't tell whether they are muddy
  - real world is 7: p2, p3, p4
- Father announce at least one is muddy:
  - m1 = convert (upd_pa initMuddy (Disj [p1,p2,p3,p4]))
    - world where all false disappears
  - aK, bK, ... state a knows its state, ...
    - all false after first round, still false after second
    - Down to only one state and done!!!

Questions?