Lecture 30: Inference

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Some slides based on those of Christina Unger

Inferences Critical

- Inferences, often using world knowledge, play a big role in understanding utterances.
  - John ate the pudding with a fork.
    John ate the pudding with vanilla flavor.
  - A: Would you like to come to the Keith Jarrett concert?
    B: I hate Jazz!

Application: Question Answering

- Was Erdös married?
  - Apart from his family and old friends, Paul Erdös had no interest in a relationship which was not founded in shared intellectual curiosity and therefore he remained a bachelor until his death.
- Did United win the Champions League?
  - United failed to progress beyond the group stages of the Champions League and trailed in the Premiership title race, sparking rumours over its future.

Types of Inferences

- Logical inferences
  - Deductive inferences
  - Inductive inferences
  - Abductive inferences
- Pragmatic inferences

Deductive Inferences

- Truth of the premises guarantees truth of the consequence, i.e. the latter necessarily follows from the former (due to form, not content).
  - If there is a unicorn in the garden, then we're in heaven. There is a unicorn in the garden. ⇒ We're in heaven.
  - If there is a unicorn in the garden, then we're in heaven. We're not in heaven. ⇒ There is no unicorn in the garden.
  - There is either a unicorn or a goblin in the garden. There is no unicorn in the garden.
    ⇒ There is a goblin in the garden.

Inductive Inferences

- The consequence does not follow necessarily from the premises, but the latter provides very good reason for inferring the former, unless there is evidence against it.
  - All stars we have ever examined burn hydrogen.
    ⇒ All stars burn hydrogen.
  - Almost all birds can fly. Dodo is a bird.
    ⇒ Dodo can fly.

Abductive Inferences

- The consequence allows the inference of the premise(s) as an explanation.
  - All birds can fly. Dodos can fly.
    ⇒ Dodos is a bird.
  - This morning my lawn was wet.
    I have no sprinklers, but every time it rains my lawn is wet.
    ⇒ It rained last night.

Pragmatic Inferences

- Implicatures are consequences drawn on the basis of general assumptions about how speakers behave in a communication.
  - Some of your books are interesting.
    ⇒ Not all of your books are interesting.
  - Yesterday John found a turtle in a garden.
    ⇒ It was not John's turtle and not John's garden.
  - John broke his hand and went to the hospital.
    ⇒ John first broke his hand and then went to the hospital.

Checking Deductive Inferences

- There are two coinciding ways for checking deductive inferences in first-order logic.
  - Semantics (via models)
    - A formula A is a semantic consequence of a set of formulas Γ iff A is true in all models in which all formulas in Γ are true.
  - Syntax (in terms of proofs)
    - A formula A is a syntactic consequence of a set of formulas Γ iff there is a formal proof deriving A from Γ.

Semantic Implication

- Let $\mathcal{M} = (M, I)$ be model, $\mathcal{M},g \models \phi$ means ϕ is true in the model.
- $\Gamma \models \phi$ *(read Γ logically implies ϕ)* iff for all $\mathcal{M}$,g,

  if $\mathcal{M},g \models \gamma$ for all $\gamma \in \Gamma$, then $\mathcal{M},g \models \phi$
- This is hard to demonstrate as requires checking all possible models of Γ.

Syntactic Implication (Proof)

- $\vdash \phi$ *(read ϕ is provable)* iff there is a proof of ϕ from the axioms of logic using a given set of inference rules.
- $\Gamma \vdash \phi$ *(read Γ proves ϕ)* iff there is a proof of ϕ that uses the formulas of Γ as hypotheses using a given set of inference rules
- This is easier to demonstrate as requires just finding a single proof object. *(But it is hard to show something is not provable.)*

Natural Deduction

- Consists of a set of rules for deriving consequences. It has an introduction and elimination rule for every logical constant.
- Used in CS 81

The basic rules of natural deduction:

	introduction	*elimination*
$\wedge$	$\dfrac{\phi \quad \psi}{\phi \wedge \psi}\wedge\mathrm{i}$	$\dfrac{\phi \wedge \psi}{\phi}\wedge\mathrm{e}_1$ $\quad \dfrac{\phi \wedge \psi}{\psi}\wedge\mathrm{e}_2$
$\vee$	$\dfrac{\phi}{\phi \vee \psi}\vee\mathrm{i}_1$ $\quad \dfrac{\psi}{\phi \vee \psi}\vee\mathrm{i}_2$	$\dfrac{\phi \vee \psi \quad \boxed{\begin{array}{c}\phi\\ \vdots\\ \chi\end{array}} \quad \boxed{\begin{array}{c}\psi\\ \vdots\\ \chi\end{array}}}{\chi}\vee\mathrm{e}$
$\rightarrow$	$\dfrac{\boxed{\begin{array}{c}\phi\\ \vdots\\ \psi\end{array}}}{\phi \rightarrow \psi}\rightarrow\mathrm{i}$	$\dfrac{\phi \quad \phi \rightarrow \psi}{\psi}\rightarrow\mathrm{e}$
$\neg$	$\dfrac{\boxed{\begin{array}{c}\phi\\ \vdots\\ \bot\end{array}}}{\neg\phi}\neg\mathrm{i}$	$\dfrac{\phi \quad \neg\phi}{\bot}\neg\mathrm{e}$
$\bot$	(no introduction rule for $\bot$)	$\dfrac{\bot}{\phi}\bot\mathrm{e}$
$\neg\neg$		$\dfrac{\neg\neg\phi}{\phi}\neg\neg\mathrm{e}$ *Classical only*

Proofs

- Ordered list of steps where each step justified as premise or by proof rule from earlier steps.
- Show $\vdash \neg(\phi \wedge \neg\phi)$

1.	$\phi \wedge \neg\phi$	*assumption*
2.	ϕ	1, $\wedge$e
3.	$\neg\phi$	1, $\wedge$e
4.	$\bot$	2,3, $\neg$e
5.	$\neg(\phi \wedge \neg\phi)$	1-4, $\neg$i

Always indicate proof rule and steps used to get new wff
Use boxes for subproofs to be discharged

Distinction between hypothesis and assumption

Unfortunately ...

- There is no straightforward implementation of Natural Deduction.
- Theorem provers therefore usually use another method for deriving consequences:
  - Tableaux *or*
  - Resolution

Resolution

- Resolution rule: $\dfrac{\alpha \vee \beta,\ \neg\beta \vee \gamma}{\alpha \vee \gamma}$
- Resolution refutation: *(proof by contradiction)*
  - Convert all sentences to conjunctive normal form
  - Negate the conclusion (in CNF)
  - Apply resolution rule until either
    - Derive false (contradiction!)
    - Can't apply it any more.
- Resolution refutation is sound & complete:
  - If contradiction then valid, else not valid

Aside

- Modus ponens is simple use of resolution:

$$\frac{P \rightarrow Q, P}{Q}$$

- is just $\dfrac{\neg P \vee Q, P}{Q}$

Example

- $P \vee Q, P \rightarrow R, Q \rightarrow R \vdash R$

Convert all to conjunctive normal form before starting

Step	*Formula*	*Derivation*
1	$P \vee Q$	Given
2	$\neg P \vee R$	Given
3	$\neg Q \vee R$	Given
4	$\neg R$	Negated conclusion
5	$Q \vee R$	1,2 - Resolution
6	$\neg Q$	3,4 - Resolution
7	R	5,6 - Resolution
8	•	4, 7 - Resolution

Strategies

- Unit preference: prefer a resolution step involving an unit clause (clause with one literal).
  - Produces a shorter clause – which is good since we are trying to produce a zero-length clause, that is, a contradiction.
- Set of support: Choose resolution involving negated goal or any clause derived from it.
  - We're trying to produce a contradiction that follows from the negated goal, so these are "relevant" clauses.
  - If a contradiction exists, one can find one using the set-of- support strategy.

Predicate Logic

- Can be extended to predicate logic using unification.
  - Express all formulas in preen form (pull all quantifiers to front) and insides in CNF.
  - Replace existential quantifiers by Skolem functions/ constants:
    - $\forall X. \exists Y.\ person(X) \rightarrow has(X, Y) \wedge heart(Y)$ *replaced by*
    - $person(X) \rightarrow has(X, f(X)) \wedge heart(f(X))$
    - Function f(X) has parameter X because ∃ inside ∀X
  - Use unification and resolution to get contradiction

Unification

- Unify following sentences:
  - $P(X,tony) \wedge Q(george, X, Z)$
  - $P(f(tony),tony) \wedge Q(B,C,maggie)$
- Substitution:
  - $X \mapsto f(tony)$, $B \mapsto george$, $C \mapsto f(tony)$, $Z \mapsto maggie$
- Use resolution, using unification to make opposites match.

Example

- Show $\forall X.\ man(X) \rightarrow mortal(X), man(socrates) \vdash mortal(socrates)$
- Rewrite in CNF:
  - $man(socrates), \neg man(X) \vee mortal(X), \neg\, mortal(socrates)$
  - Unify X and socrates:
    - $man(socrates), \neg man(X) \vee mortal(X), \neg\, mortal(socrates), \neg man(socrates) \vee mortal(socrates)$
  - Use resolution on first & last:
    - $man(socrates), \neg man(X) \vee mortal(X), \neg\, mortal(socrates), \neg man(socrates) \vee mortal(socrates), mortal(socrates)$
  - Get contradiction from $\neg mortal(socrates), mortal(socrates)$

Tableaux Method

Tableaux Method

- Similar to resolution, in based on finding contradictions.
- Set up proof tree, where path represents an alternative to making formula true. Branches indicate where there are alternatives.
- No need to convert to cnf or prenex form.

Rules

	Affirmed	*Denied*
Not	$\frac{\neg\phi \quad \phi}{x}$	$\frac{\neg\neg\phi}{\phi}$
And	$\frac{\phi \wedge \psi}{\phi \quad \psi}$ (one below the other)	$\frac{\neg(\phi \wedge \psi)}{\neg\phi \quad \neg\psi}$
Or	$\frac{\phi \vee \psi}{\phi \quad \psi}$	$\frac{\neg(\phi \vee \psi)}{\neg\phi \quad \neg\psi}$ (one below the other)
Implies	$\frac{\phi \to \psi}{\neg\phi \quad \psi}$	$\frac{\neg(\phi \to \psi)}{\phi \quad \neg\psi}$ (one below the other)

Rules

	Affirmed	*Denied*
All	$\frac{\forall x.\ \ldots x \ldots}{\ldots n \ldots}$ *for any n*	$\frac{\neg\forall x.\ \ldots x \ldots}{\exists x.\ \neg \ldots x \ldots}$
Exists	$\frac{\exists x.\ \ldots x \ldots}{\ldots c \ldots}$ *for new c*	$\frac{\neg\exists x.\ \ldots x \ldots}{\forall x.\ \neg \ldots x \ldots}$

Example

Prove: $(\forall x)(T(x,b) \to T(a,b)),\ \neg T(a,b) \vdash \neg(\exists x)T(x,b)$

1. $(\forall x)(T(x,b) \to T(a,b))$ (premise)
2. $\neg T(a,b)$ (premise)
3. $\neg\neg(\exists x)T(x,b)$ (premise)
4. $(\exists x)T(x,b)$ (3, Not denied)
5. $T(c,b)$ (4, Exist affirmed)
6. $T(c,b) \to T(a,b)$ (1, All affirmed)

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7. $\neg T(c,b)$ $\quad$ $T(a,b)$ (1, Imply affirmed)
8. x $\quad$ x (2,5,7, Not affirmed)

Questions?