Lecture 30: Inference

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Some slides based on those of Christina Unger

Inferences Critical

- Inferences, often using world knowledge, play a big role in understanding utterances.
 - John ate the pudding with a fork. John ate the pudding with vanilla flavor.
 - A: Would you like to come to the Keith Jarrett concert?
 B: I hate Jazz!

Application: Question Answering

- Was Erdös married?
 - Apart from his family and old friends, Paul Erdös had no interest in a relationship which was not founded in shared intellectual curiosity and therefore he remained a bachelor until his death.
- Did United win the Champions League?
 - United failed to progress beyond the group stages of the Champions League and trailed in the Premiership title race, sparking rumours over its future.

Types of Inferences

• Logical inferences

- Deductive inferences
- Inductive inferences
- Abductive inferences
- Pragmatic inferences

Deductive Inferences

- Truth of the premises guarantees truth of the consequence, i.e. the latter necessarily follows from the former (due to form, not content).
 - If there is a unicorn in the garden, then we're in heaven.
 There is a unicorn in the garden. ⇒ We're in heaven.
 - If there is a unicorn in the garden, then we're in heaven.
 We're not in heaven. ⇒ There is no unicorn in the garden.
 - There is either a unicorn or a goblin in the garden. There is no unicorn in the garden.
 ⇒ There is a goblin in the garden.

Inductive Inferences

- The consequence does not follow necessarily from the premises, but the latter provides very good reason for inferring the former, unless there is evidence against it.
 - All stars we have ever examined burn hydrogen.
 ⇒ All stars burn hydrogen.
 - Almost all birds can fly. Dodo is a bird.
 ⇒ Dodo can fly.

Abductive Inferences

- The consequence allows the inference of the premise(s) as an explanation.
 - All birds can fly. Dodos can fly.
 ⇒ Dodos is a bird.
 - This morning my lawn was wet. I have no sprinklers, but every time it rains my lawn is wet.
 - \Rightarrow It rained last night.

Pragmatic Inferences

- Implicatures are consequences drawn on the basis of general assumptions about how speakers behave in a communication.
 - Some of your books are interesting.
 ⇒ Not all of your books are interesting.
 - Yesterday John found a turtle in a garden.
 ⇒ It was not John's turtle and not John's garden.
 - John broke his hand and went to the hospital.
 ⇒ John first broke his hand and then went to the hospital.

Checking Deductive Inferences

- There are two coinciding ways for checking deductive inferences in first-order logic.
 - Semantics (via models)
 - A formula A is a semantic consequence of a set of formulas Γ iff A is true in all models in which all formulas in Γ are true.
 - Syntax (in terms of proofs)
 - A formula A is a syntactic consequence of a set of formulas Γ iff there is a formal proof deriving A from $\Gamma.$

Semantic Implication

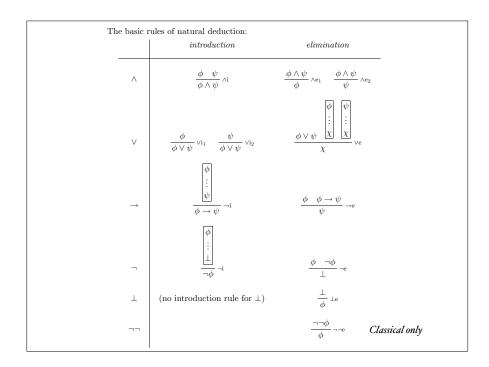
- Let *M* = (M, I) be model, *M*,g ⊨ φ means φ is true in the model.
- $\Gamma \vDash \varphi$ (read Γ logically implies φ) iff for all \mathcal{M},g , if $\mathcal{M},g \vDash \gamma$ for all $\gamma \in \Gamma$, then $\mathcal{M},g \vDash \varphi$
- This is hard to demonstrate as requires checking all possible models of Γ.

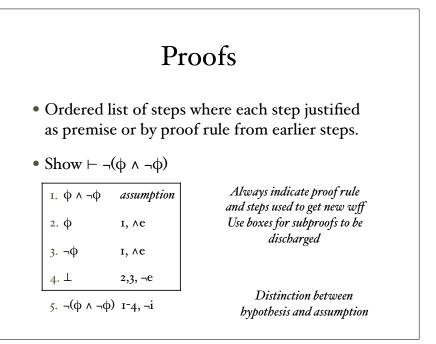
Syntactic Implication (Proof)

- ⊢ φ (*read* φ *is provable*) iff there is a proof of φ from the axioms of logic using a given set of inference rules.
- Γ⊢ φ (read Γ proves φ) iff there is a proof of φ that uses the formulas of Γ as hypotheses using a given set of inference rules
- This is easier to demonstrate as requires just finding a single proof object. (But it is hard to show something is not provable.)

Natural Deduction

- Consists of a set of rules for deriving consequences. It has an introduction and elimination rule for every logical constant.
- Used in CS 81





Unfortunately ...

- There is no straightforward implementation of Natural Deduction.
- Theorem provers therefore usually use another method for deriving consequences:
 - Tableaux or
 - Resolution

Resolution

- Resolution rule: $\frac{\alpha \lor \beta, \neg \beta \lor \gamma}{\alpha \lor \gamma}$
- Resolution refutation: (proof by contradiction)
 - Convert all sentences to conjunctive normal form
 - Negate the conclusion (in CNF)
 - Apply resolution rule until either
 - Derive false (contradiction!)
 - Can't apply it any more.
- Resolution refutation is sound & complete:
 - If contradiction then valid, else not valid

Aside

• Modus ponens is simple use of resolution:

$$\frac{P \rightarrow Q, P}{Q}$$

• is just

<u>--P v Q, P</u> O

Example

•
$$P \lor Q, P \rightarrow R, Q \rightarrow R \vdash R$$

	Step	Formula	Derivation
Convert all to	I	ΡvQ	Given
conjunctive	2	¬ P v R	Given
normal form	3	¬ Q v R	Given
before starting	4	¬R	Negated conclusion
	5	Q v R	1,2 - Resolution
	6	¬Q	3,4 - Resolution
	7	R	5,6 - Resolution
	8	•	4, 7 - Resolution

Strategies

- Unit preference: prefer a resolution step involving an unit clause (clause with one literal).
 - Produces a shorter clause which is good since we are trying to produce a zero-length clause, that is, a contradiction.
- Set of support: Choose resolution involving negated goal or any clause derived from it.
 - We're trying to produce a contradiction that follows from the negated goal, so these are "relevant" clauses.
 - If a contradiction exists, one can find one using the setof- support strategy.

Predicate Logic

- Can be extended to predicate logic using unification.
 - Express all formulas in preen form (pull all quantifiers to front) and insides in CNF.
 - Replace existential quantifiers by Skolem functions/ constants:
 - $\forall X. \exists Y. person(X) \rightarrow has(X, Y) \land heart(Y)$ replaced by
 - $person(X) \rightarrow has(X, f(X)) \land heart(f(X))$
 - + Function f(X) has parameter X because \exists inside $\forall X$
 - Use unification and resolution to get contradiction

Unification

- Unify following sentences:
 - $P(X,tony) \land Q(george, X, Z)$
 - P(f(tony),tony) ^ Q(B,C,maggie)
- Substitution:
 - $X \mapsto f(tony), B \mapsto george, C \mapsto f(tony), Z \mapsto maggie$
- Use resolution, using unification to make opposites match.

Example

- Show ∀X. man(X) → mortal(X), man(socrates)
 ⊢ mortal(socrates)
- Rewrite in CNF:
 - man(socrates), ¬man(X) v mortal(X), ¬ mortal(socrates)
 - Unify X and socrates:
 - man(socrates), ¬man(X) v mortal(X), ¬ mortal(socrates), ¬man(socrates) v mortal(socrates)
 - Use resolution on first & last:
 - man(socrates), ¬man(X) v mortal(X), ¬ mortal(socrates), ¬man(socrates) v mortal(socrates), mortal(socrates)
 - Get contradiction from ¬mortal(socrates), mortal(socrates)

Tableaux Method

Tableaux Method

- Similar to resolution, in based on finding contradictions.
- Set up proof tree, where path represents an alternative to making formula true. Branches indicate where there are alternatives.
- No need to convert to cnf or prenex form.

	Rules	
	Affirmed	Denied
Not	¬φ _φ x	<u>φ</u> φ
And	<u>φ ∧ ψ</u> φ ψ	<u>-(φ ^ ψ)</u> -φ -ψ
Or	<u>φνψ</u> φψ	<u>(φ v ψ)</u> φ ψ
Implies	$\frac{\phi \rightarrow \psi}{\neg \phi \psi}$	$\begin{array}{c} \underline{\neg(\phi \rightarrow \psi)} \\ \phi \\ \neg\psi \end{array}$

	Affirmed	Denied
All	∀xx n for any n	¬ ∀xx ∃x. ¬x
Exists	<u>∃xx</u> c for new c	<u>_∃xx</u> ∀xx

Example Prove: $(\forall x)(T(x,b) \rightarrow T(a,b)), \neg T(a,b) \vdash \neg (\exists x)T(x,b)$ 1. $(\forall x)(T(x,b) \rightarrow T(a,b))$ (premise) 2. ¬T(a,b) (premise) Questions? 3. ¬¬(∃x)T(x,b) (premise) 4. $(\exists x)T(x,b)$ (3, Not denied) 5. T(c,b) (4, Exist affirmed) 6. $T(c,b) \rightarrow T(a,b)$ (I, All affirmed) / 7. ¬T(c,b) T(a,b) (I, Imply affirmed) 8. (2,5,7, Not affirmed) х х