Lecture 22: Intensional Logic

CS 181O
Spring 2016
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Extension vs Intension

- Opaque contexts
  - Eight is necessarily greater than seven.
    The number of planets in our solar system is necessarily greater than seven.
  - I believe that Jane talked to the President of Pomona.
    I believe that Jane talked to David Oxtoby.
  - Mary is looking for the President of Pomona.
    Mary is looking for David Oxtoby.
  - Since those sentences mean different things, meaning seems to be more than reference.
  - Truth depends on possible as well as actual worlds.

Intensional Operators

- Intensional models help interpret
  - adjectives like "fake", "former",
  - attitude verbs like "want", "hope"
  - "must", "may", "necessarily", "possibly"

Intensional Propositional Logic

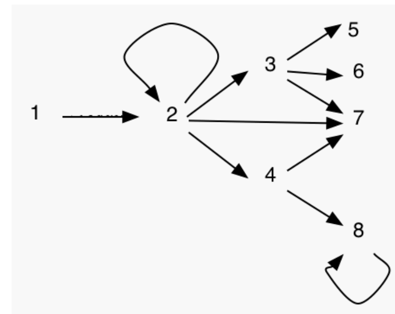
- If p is a proposition letter, then p is a formula
- If ϕ and ψ are formulas then so are $\phi\wedge\psi$, $\phi\vee\psi$, $\phi\rightarrow\psi$, $\neg\phi$, and $O\phi$.
  - Meaning of $O\phi$ will depend on the set of contexts that we are interested in
    - Necessarily ϕ, always in the future ϕ, ...

Models for Intensional Logic

- A (Kripke) model M consists of
  - a non-empty set W of contexts,
  - a binary relation R on W, the accessibility relation
  - A valuation function V which assigns a truth value $V_w(p)$ to every proposition letter p in each context w.
- Contexts referred to as possible worlds
- Combination of W,R called a "frame"

Contexts & Accessibility

- Accessibility relation:
  - R = {(v1, v2), (v2, v2), (v2, v3), (v2, v4), (v2, v7), (v3, v5), (v3, v6), (v3, v7), (v4, v7), (v4, v8), (v8, v8)}
  - or



Truth in Intensional Propositional Logic

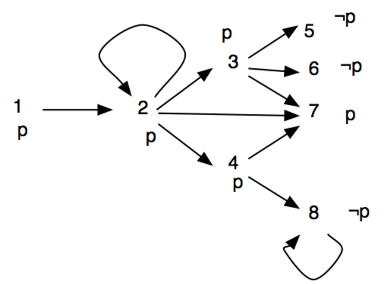
- Let M be model with W as set of possible worlds, R as accessibility relation, and V as valuation, then $V_{M,w}(\phi)$, the truth value of ϕ in w given M is defined as follows:
- $V_{M,w}(p) = V_w(p)$ for all proposition letters p.
- $V_{M,w}(\neg\phi)$ = true iff $V_w(\phi)$ = false.
- $V_{M,w}(\phi \rightarrow \psi)$ = true iff
  $V_{M,w}(\phi)$ = false or $V_{M,w}(\psi)$ = true.
- ...
- $V_{M,w}(O\phi)$ = true iff $\forall w' \in W$ s.t. $\langle w,w' \rangle \in R$,
  $V_{M,w'}(\phi)$ = true.

Modal Logic: Necessity

- Replace $O\phi$ by $\Box\phi$, dual $\Diamond\phi \equiv \neg\Box\neg\phi$
  - $\Box\phi$ means "necessarily ϕ"
  - $\Diamond\phi$ means "possibly ϕ"
  - If ϕ stands for "you understand me", then translate: "It is possible that you understand me, but it isn't necessary" as $\Diamond\phi \wedge \neg\Box\phi$

Interpreting Necessity

- Label worlds with p or ¬p



Which worlds satisfy $\Box p$, $\Diamond p$?

What about $\Box\Diamond p$, $\Box\Box p$?

Structure & Truth

- With which accessibility relations are the following expressions always true?
  - $\Box\phi \rightarrow \phi$
  - $\Box(\phi \rightarrow \psi) \rightarrow (\Box\phi \rightarrow \Box\psi)$
  - $\Box\Box\phi \rightarrow \Box\phi$
  - $\Diamond\Box\phi \rightarrow \phi$

Other Modal Operators

- Operations on sentences:
  - must, may
    - lots of different senses, eg. "you must drive the speed limit" vs 1+1 must equal 2
  - knows
  - wants
  - hope
- Can also reason about time with modal operators:
  - always in the future, at some time in the future

Counterfactuals & Possible Worlds

- *If I didn't come to Pomona, you would not be taking this course.*
- *If I didn't come to Pomona, you would be crying.*
- Why is the first more plausible, even though they are both true in propositional logic?

Interpretations of Frames

- Interpreting "must" ("may is the dual")
  - Epistemic: w Epi_x w' iff w' conforms to what x knows in w.
  - Deontic: w Deo w' iff all the obligations are fulfilled in w' and w' is maximally similar to w otherwise.
  - w Dox_x w' iff w' conforms to what x believes in w to be the case.
  - Bouletic: w Bou_x w' iff w' conforms to what x desires in w for it to be the case

Types

- Let s be type for set of worlds, with e and t as before. s can only be used as a domain of functions.
- The set of all types, T, is the smallest set such that
  - $e, t \in T$
  - if $a, b \in T$ then so is $a \rightarrow b$
  - if $a \in T$, then so is $s \rightarrow a$

Predicate Modal Logic

- Add $\Box\phi$, $\Diamond\phi$ to predicate logic as well as

  terms $^\wedge\phi$ and $^\vee\phi$ to represent the intension of ϕ and interpreting ϕ in the current world.

Predicate Modal Logic

- Add $\Box$, $\Diamond$ to predicate logic.
- Definition. A model consists of
  - a non-empty set W of worlds
  - A binary accessibility relation R on W.
  - A domain D
  - An interpretation function I which assigns
    - an entity I(c) to each constant c of L,
    - A non-empty subset $I_w(E)$ of D to E for each world w in W, and
    - for every world w in W, a subset $I_w(R)$ of D_w^n to each n-ary predicate symbol R of L.

See model in EAI.hs

- IBool = World → Bool
- IEntity = World → Entity
- iSent :: Sent → IBool
- iNP :: NP → (IEntity → IBool) → IBool
- iVP :: VP → (IEntity → IBool)
- iCN :: CN → (IEntity → IBool)
- iDet :: DET → (IEntity → IBool) →
  (IEntity → IBool) → IBool

Questions?