Lecture 13: Semantics of Predicate Logic & Generalized Quantifiers

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Some slide content taken from Unger and Michaelis

Semantics of Predicate Logic

- Now ready to show interpretations in a model.
- See file Model.hs (and Model2.hs) for examples of models of language in FSynF.hs
  - D = {A,B,C,...,Z,Unspec}
    - Because declared as Bounded, can refer to as [minBound..maxBound]
  - Associate constants with elements of D (= Entity)

Model Encoding

- Includes functions to convert from lists to one-place characteristic functions (i.e., for unary relations)
  - Characteristic functions for binary and ternary relations are Curried (e.g., Entity -> Entity -> Bool)
  - *Ignore passivize and self for now.*

Semantics of Predicate Logic

- Interpretation Functions defined
  - int_0 takes relation name and list of entities and returns value (according to Model)
  - LookUp is type of assignments of values to variables
    - change update a variable assignment
  - Function eval takes a domain (list of elts), an interpretation of relational symbols, a variable assignment, and a formula and tells whether true or false.
    - *eval does NOT handle functions, the only terms are variables!!! More later*
    - Helper function eval' uses fixed model and interp of relations

From MCWPL.hs

Including Terms

- Function of type FInterp takes function name and list of args (from domain) and returns value in domain of model (*see fint1 plus*)
- liftLookUp takes assignment of meaning to function expressions, assignment of values to variables and a term, and returns its value.
- eval takes model domain, interpretation of relation symbols, interp of function symbols, & assignment of value to variables and formula and returns whether true or false.

Evaluating formulas

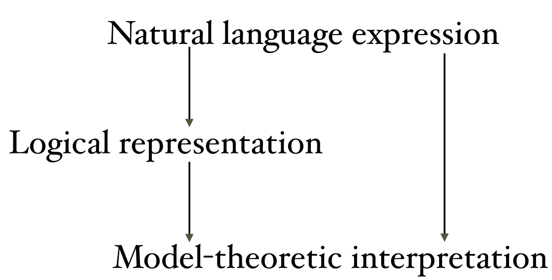
- After loading MCWPL:

```
*MCWPL> evl entities int3 fint2 ass1 formula3
False

*MCWPL> evl entities int3 fint2 ass1 formula4
True
```

Interpreting language

- Two options: indirect & direct



Done!

Why do the second?

- Principle of compositionally:
  - Meaning of whole composed from meaning of parts
  - Want to preserve structure of sentences.
  - Every girl liked a dog
    - $\forall x\ (girl(x) \rightarrow \exists y(dog(y) \wedge liked(x,y)))$
    - *Draw parse trees!*
- Problems with general quantifiers

Generalized Quantifiers

- Approach due to Barwise & Cooper (1981)
- Quantifiers are binary relations over power set of domain of discourse.
  - Every dog barked: $\{x \mid dog(x)\} \subseteq \{x: barked(x)\}$
  - A dog barked: $\{x \mid dog(x)\} \cap \{x: barked(x)\} \neq \emptyset$
  - Most dogs barked:
    $|\{x \mid dog(x)\} \cap \{x: barked(x)\}| > 0.5 * |\{x \mid dog(x)\}$

Conditions on Quantifiers

- Write $D_E AB$ to stand for determiner expression (like those on previous slide) with E the domain of discourse, A the restriction and B its body.
  - E.g., "Every dog barked" has dog(x) as restriction and barked(x) as the body.
  - Similarly for "A dog barked" or "Most dogs barked"

Conditions on Quantifiers

- Require:
  - EXT: For all A, $B \subseteq E \subseteq E'$, $D_E AB \Leftrightarrow D_{E'} AB$
    - *Extension*
    - Expanding the domain makes no difference to truth if A, B fixed.
    - Really, only $A \cup B$ matters
  - CONS: For all A, $B \subseteq E \subseteq E'$, $D_E AB \Leftrightarrow D_{E'} A(A \cap B)$
    - *Conservativity*
    - For the body, only the elements in the body matter
    - Not hold of "Only dogs barked"
  - EXT + CONS $\Rightarrow$ Only A-B and $A \cap B$ matter in determining truth of $D_E AB$

Expressing Quantifiers

- Quantifiers can be expressed using only $|A \cap B|$ and $|A - B|$
  - All A are B $\Rightarrow |A - B| = 0$
  - Some A are B $\Rightarrow |A \cap B| > 0$
  - Most A are B $\Rightarrow |A \cap B| > |A - B|$

Further Conditions

- For quantifiers on quantity:
  - ISOM: If f is a bijection from E to E',
    then $D_E AB \Leftrightarrow D_{E'} f[A]\, f[B]$

Questions?