Homework 1

Due Thursday, 01/28/2016, by midnight

You will find it most helpful to prepare your solutions using LaTeX.

Your homework should be turned in via Sakai. Go to sakai.claremont.edu and log in. Click on the header PO CSCI 181O.1 SP16. In the left margin select Assignments. Click on the correct assignment number and then follow the directions to turn in your solution.

0. If you do not know LaTeX, please read the on-line documentation so that you can understand how the LaTeX version of this (the ".tex" file) generated this document. (*Do not turn in anything for this part.*)

1. (10 points) Let P be the child binary relation = {(c,p) | c is a child of p}.
   (a) Please give an intuitive explanation for the relation $P \circ P$.
   (b) Please give an intuitive explanation for the reverse of P, P^{-}.

2. (9 points) The operation = is an equivalence relation. That is, it is reflexive, symmetric, and transitive.

   Consider the operation $\neq$. Determine whether it is reflexive, symmetric, and/or transitive. Explain each of your your answers. If the answer is no, give a counter-example. If it is true, provide intuitive reasons why it is so.

3. (6 points) In class we defined the characteristic function of a set A to be the function $f_A : A \rightarrow Bool$ s.t. $f_A(a) =$ true iff $a \in A$.

   if f_A is the characteristic function for A, we can define the characteristic function for the complement of A, $\bar{A}$ by writing $f_{\bar{A}}(a) = \neg f_A(a)$ for all a. Thus we can model a set-theoretic definition (of complement) with a logical operation (negation).

   Given characteristic functions for sets A and B, give similarly simple definitions for the characteristic functions for $A \cup B$ and $A \cap B$ by using logical operations (e.g. $\vee$ and $\wedge$ on the values of the characteristic functions for A and B.

4. (10 points) For each variable occurrence in the following terms of the untyped lambda calculus, identify that variable as either bound or free. If it is bound identify the lambda that binds it.
   (a) $(\lambda x.x)(\lambda y.x)(\lambda x.x\ y)$
   (b) $(\lambda x.x\ z)((\lambda y.y\ w)(\lambda w.w\ y\ z\ x))$

5. (10 points) Apply β reduction to the following lambda expressions as much as possible. Use α reduction as needed to avoid variable capture. (In the examples, interpret ++ as string concatenation and the strings "jane" and "hit the ball" as constants.
   (a) $(\lambda j.\lambda v.v\ j) jane (\lambda p.p{+}{+}hit\ the\ ball)$
   (b) $(\lambda x.\lambda y.x\ y\ y)(\lambda a.a)b$