

Restrictions on ML/Haskell Polymorphism

- Type $(a \rightarrow b) \rightarrow ([a] \rightarrow [b])$ stands for:
 - $\forall a. \forall b. (a \rightarrow b) \rightarrow ([a] \rightarrow [b])$
- Haskell functions may not take polymorphic arguments. E.g., no type:
 - $\forall b. ((\forall a.(a \rightarrow a)) \rightarrow (b \rightarrow b))$
 - define: `foo f (x,y) = (f x, f y)` ;
 - `id z = z`
 - `foo id (7, True)` -- gives type error!
 - Type of `foo` is only $(t \rightarrow s) \rightarrow (t, t) \rightarrow (s, s)$

Restrictions on Implicit Polymorphism

Polymorphic types can be defined at top level or in let clauses, but can't be used as arguments of functions

```
id x = x
in (id "ab", id 17)
```

OK, but can't write

```
test g = (g "ab", g 17)
```

Can't find type of `test` w/unification.
More general type inference is undecidable.

Explicit Polymorphism

Easy to type w/ explicit polymorphism:

```
test (g: forall t.t -> t) = (g "ab", g 17)
in test (\ t => \ (x:t) -> x)
```

Languages w/explicit polymorphism:

Clu, Ada, C++, Eiffel, Java 5, C#, Scala, Grace

Explicit Polymorphism

- Clu, Ada, C++, Java
- C++ macro expanded at link time rather than compile time.
- Java compiles away polymorphism, but checks it statically.
- Better implementations keep track of type parameters.

Summary

- Modern tendency: strengthen typing & avoid implicit holes, but leave explicit escapes
- Push errors closer to compile time by:
 - Require over-specification of types
 - Distinguishing between different uses of same type
 - Mandate constructs that eliminate type holes
 - Minimizing or eliminating explicit pointers
- Holy grail: Provide type safety, increase flexibility

Polymorphism vs Overloading

- Parametric polymorphism
 - Single algorithm may be given many types
 - Type variable may be replaced by any type
 - Examples: `hd, tail :: [t] -> t` , `map :: (a -> b) -> [a] -> [b]`
- Overloading
 - A single symbol may refer to more than one algorithm.
 - Each algorithm may have different type.
 - Choice of algorithm determined by type context.
 - + has types `Int -> Int -> Int` and `Float -> Float -> Float`, but not `t -> t -> t` for arbitrary `t`.

Why Overloading?

- Many useful functions not parametric
 - List membership requires equality
 - `member: [w] -> w -> Bool` (for "good" `w`)
 - Sorting requires ordering
 - `sort: [w] -> [w]` (for `w` supporting `<, >, ...`)
- What are problems in supporting it in a PL?
 - Static type inference makes it hard!
 - Why are Haskell type classes a solution?

Overloading Arithmetic

- First try: allow fcns w/overloaded ops to define multiple functions
 - `square x = x * x`
 - versions for `Int -> Int` and `Float -> Float`
 - But then
 - `squares (x,y,z) = (square x, square y, square z)`
 - ... has 8 different versions!!
 - Too complex to support!

ML & Overloading

- Functions like +, * can be overloaded, but not functions defined from them!
 - `3 * 3` -- legal
 - `3.14 * 3.14` -- legal
 - `square x = x * x` -- Int -> Int
 - `square 3` -- legal
 - `square 3.14` -- illegal
- To get other functions, must include type:
 - `squaref (x:float) = x * x` -- float -> float

Equality

- Equality worse!
 - Only defined for types not containing functions, files, or abstract types -- *why?*
 - Again restrict functions using ==
- ML ended up defining eq types, with special mark on type variables.
 - member: "a -> [a] -> Bool"
 - Can't apply to list of functions

Type Classes

- Proposed for Haskell in 1989.
- Provide concise types to describe overloaded functions -- avoiding exponential blow-up
- Allow users to define functions using overloaded operations: +, *, <, etc.
- Allow users to declare new overloaded functions.
 - Generalize MLs eqtypes
 - Fit within type inference framework

Recall ...

- Definition of quicksort & partition:

```
partition /IThan (pivot, []) = ([], [])
partition /IThan (pivot, first : others) =
  let
    (smalls, bigs) = partition /IThan (pivot, others)
  in
    if (/IThan first pivot)
    then (first:smalls, bigs)
    else (smalls, first:bigs)
```
- Allowed partition to be parametric
 - Steal this idea to pass overloaded functions!
 - Implicitly pass argument with any overloaded functions needed!!

Example

- Recall

```
class Order a where
  (<) :: a -> a -> Bool
  (>) :: a -> a -> Bool
  ...
```

- Implement w/dictionary:

```
data OrdDict a = MkOrdDict (a -> a -> Bool) (a -> a ->
Bool) ...

getLT (MkOrdDict lt gt ...) = lt
getGT (MkOrdDict lt gt) = gt
```

Using Dictionaries

```
partition dict (pivot, []) = ([],[])
partition dict (pivot, first : others) =
  let
    (smalls, bigs) = partition dict (pivot, others)
  in
    if ((getLT dict) first pivot)
    then (first:smalls, bigs)
    else (smalls, first:bigs)
```

```
partition:: OrdDict a -> [a] -> ([a].[a])
```

Compiler adds dictionary parameter to all calls of partition.

Reports type of partition (w/out lThan parameter) as
(Ord a) => [a] -> ([a].[a])

Instances

- Declaration

- instance Show TrafficLight where
 show Red = "Red light"
 show Yellow = "Yellow light"
 show Green = "Green light"
- Creates dictionary for "show"

Implementation Summary

- Compiler translates each function using an overloaded symbol into function with extra parameter: the *dictionary*.
- References to overloaded symbols are rewritten by the compiler to lookup the symbol in the *dictionary*.
- The compiler converts each type class declaration into a *dictionary* type declaration and a set of *selector* functions.
- The compiler converts each instance declaration into a *dictionary* of the appropriate type.
- The compiler rewrites calls to overloaded functions to pass a *dictionary*. It uses the static, qualified type of the function to select the dictionary.

Multiple Dictionaries

- Example:
 - squares :: (Num a, Num b, Num c) => (a, b, c) -> (a, b, c)
 - squares(x,y,z) = (square x, square y, square z)
- goes to:
 - squares (da,db,dc) (x, y, z) =
 (square da x, square db y, square dc z)

Compositionality

- Build compounds from simpler
 - class Eq a where
 (==) :: a -> a -> Bool
 - instance Eq Int where
 (==) = intEq -- *where intEq primitive equality*
 - instance (Eq a, Eq b) => Eq(a,b) where
 (u,v) == (x,y) = (u == x) && (v == y)
 - instance Eq a => Eq [a] where
 (==) [] [] = True
 (==) (x:xs) (y:ys) = x==y && xs == ys
 (==) _ _ = False

Subclasses

- Example:
 - class (Eq a) => Num a where
 (+) :: a -> a -> a
 ... *other arith ops*
 fromInteger :: Integer -> a
 - instance of Num a must be Eq a
 - dictionary for Eq is part of that for Num

What about Literals?

- fromInteger in Num class makes it possible
 - data Complex a = MkCmpx a a *deriving Eq*
 - instance Show a => Show (Complex a) where
 show (MkCmpx rv iv) = (show rv)++" + "++ (show iv)++"i"
 - instance Num a => Num (Complex a) where
 (MkCmpx r1 i1) + (MkCmpx r2 i2) =
 MkCmpx (r1+r2) (i1+i2)
 ...
 fromInteger n = MkCmpx (fromInteger n) 0
 - fromInteger will be called implicitly when needed