Homework 12

Due Wednesday, 5/1/2019

Please submit your homework solutions on Gradescope as usual. If you have more than one file to be turned in, please put it in a folder and zip it up before turning it in.

1. (0 points) **Academic Honesty**

2. (10 points) **Computable Functions and semi-decidability**

   (a) Show that a set L is semi-decidable iff it is the domain of a Turing-computable (partial) function. That is, L is semi-decidable iff there is a Turing-computable function f such that $L = \{x | f(x) halts\}$.

   Hint: ($\Rightarrow$) Use the TM accepting L to define a function whose domain is L. Note that it doesn't matter what the function returns as a value. Reverse the hint for the other direction.

   (b) Show that a set L is semi-decidable iff it is the range of a Turing-computable (partial) function. That is, L is semi-decidable iff there is a Turing-computable function g such that $L = \{y | there is an x such that g(x) = y\}$.

   Hint: ($\Rightarrow$) Use the dovetailing technique to compute a function. If you try hard enough you can actually show (for two points extra credit) that a semi-decidable set L is either finite or the range of an injective total Turing computable function. For another point of extra credit show a semi-decidable set L is either empty or the range of a total Turing computable function.

3. (20 points) **More Diagonalization**

   Let $TOTAL_{\mathrm{TM}} = \{\langle M \rangle \mid M$ is a Turing machine that halts on *all* inputs$\}$. In class we showed that $TOTAL_{TM}$ is not decidable. Prove that neither $TOTAL_{\mathrm{TM}}$ nor its complement is semi-decidable.

   Hints: Recall that a language is decidable if and only if both it and its complement are semi-decidable.

   For $TOTAL_{\mathrm{TM}}$, assume that it is the range of a computable function (see the previous problem) and then diagonalize to obtain a contradiction. Then show that if the complement of $TOTAL_{\mathrm{TM}}$ is semi-decidable, then so is the complement of H_{TM}.

4. (10 points) **Programming language undecidability**

   Problem 21.7c in Rich, page 484

5. (10 points) **CFG undecidability**

   Problem 22.7c in Rich, page 509.