Homework 11

Due Wednesday, 4/25/2019

Purpose:

The purpose of this homework assignment is to help you better understand decidability and semi-decidability.

Turn in your Hmwk11.pdf file to gradeScope as usual. There are no programs in this assignment.

In problems where you are asked to show a problem is decidable or semi-decidable, an informal explanation of your solutions is fine. You need not show a TM program as your solution. Instead give a description of the solution where each step is clearly effectively computable.

1. (0 points) **Academic Honesty**

2. (10 points) **Decidability**

   Recall that L^* is defined as $\{w_1 \dots w_n \mid \forall 1 \leq i \leq n, w_i \in L\}$

   (a) Assume L is a decidable language. Show that L^* is also decidable.

   (b) Assume L is a semi-decidable language. Show that L^* is also semi-decidable.

3. (5 points) **Trick question!**

   Is there a Turing machine M satisfying the following condition?

   > The machine M halts on each input word w, and it accepts the word w if and only if there is a proof of $P = NP$.

   - Why was this a trick question?

4. (10 points) **Decidability**

   A language L is *co-semi-decidable* if its complement $\overline{L}$ is semi-decidable. Let A and B be disjoint co-semi-decidable languages. Show that there is a decidable language S such that $A \subseteq S$ and $B \subseteq \overline{S}$. Hint: Think about the value of $\overline{A} \cup \overline{B}$ and how you can use that to get a decidable language satisfying the given constraints.

5. (10 points) **Reduction**

   Prove that the following question is undecidable. Given a Turing machine M, is $L(M)$ finite? (Do not use Rice's Theorem in your proof.)

6. (10 points) **Reduction**

   One of the following sets is semi-decidable, and the other is not. Which is which? Give proofs for both.

   (a) $\{M \mid L(M)$ contains at least 481 elements$\}$

   (b) $\{M \mid L(M)$ contains at most 480 elements$\}$

   Hint: Recall that a set is decidable if and only if both it and its complement are semi-decidable. Use the contrapositive to show one of these is not semi-decidable. (Do not use Rice's Theorem in your proof.)