Homework 02

Due Wednesday, Feb 6

Purpose:

The purpose of this homework assignment is to help you comprehend the concepts and properties of regular languages. In this assignment, we will also go beyond just asking you to apply an algorithm to solve a problem; you will practice two skills that are at the core of CS101 and the field of computer science in general: how to formulate a computing problem, then solve it. (see question 4).

Tasks:

**Problems from the texts are given in the form c.n where c is the chapter and n is the problem number. Thus problem 2.7 is problem 7 from Chapter 2.*

1. (0 points) **Academic Honesty Declaration** (1 page)
2. (6 points) **NDFSM → DFSM** (two pages for each question)

   Problem 5.9 (a, c) from Rich
3. (5 points) **Closure** (1 page for each sub-question)

   Problem 8.7(c,d) from Rich. You may use any of the representations of regular languages discussed in class. Do provide an argument as to why your construction gives all the strings of the desired language and no extras.
4. (5 points) **Regular Languages** (2 pages)

   Let $\Sigma_3 = \left\{ \begin{bmatrix}0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}, \ldots, \begin{bmatrix}1\\1\\1\end{bmatrix} \right\}$.

   The alphabet Σ_3 has eight letters, consisting of all the size-three columns of 0's and 1's. Consider each row to be a binary number, and let

   $$B = \{w \mid \text{the bottom row is the sum of the two upper rows}\}.$$

   For example, $\begin{bmatrix}0\\0\\1\end{bmatrix}\begin{bmatrix}1\\0\\0\end{bmatrix}\begin{bmatrix}1\\1\\0\end{bmatrix} \in B$, but $\begin{bmatrix}0\\0\\1\end{bmatrix}\begin{bmatrix}1\\0\\1\end{bmatrix} \notin B$, Show that B is regular.1

   Hint: Working with $B^{\mathcal{R}}$ is easier. You may use the result of problem 3.
5. (3 points) **Minimizing FSMs** (2 pages)

   Problem 5.12 from Rich
6. (2 points) **More Closure** (1 page)

   In this problem, I want you to show the reverse of problem 8.7d.

   Let Σ, Δ be alphabets and $h : \Sigma \to \Delta^*$. We can extend h to Σ^* as follows:

   $h(\epsilon) = \epsilon$
   $h(wa) = h(w)h(a)$ for any $w \in \Sigma^*, a \in \Sigma$

   Any function $h : \Sigma^* \to \Delta^*$ defined in this way from a function $h : \Sigma \to \Delta$ is called a homomorphism.

Let $h : \Sigma^* \to \Delta^*$ be a homomorphism. Show that if $L \subset \Delta^*$ is regular (i.e., accepted by a finite state machine) then so is $\{w \in \Sigma^* | h(w) \in L\}$.

Hint: Start from a deterministic finite state machine M accepting L and construct one with the same states which, when it reads an input symbol a delivers $h(a)$ *in its imagination* to M for processing.

Criteria:

Your assignment will be graded based on the correctness and the completeness of your solutions. Please make sure that you not only provide the solutions, but also

- the rational behind it using correct, formal arguments;
- the intermediate steps and results.

Submission Guideline:

Please edit your HW following the editing guideline, especially follow the instructions on the number of pages each question should occupy. The text in blue are instructions; they should be either removed, or replaced with proper contents and turned into black. Please submit your homework solutions online via gradescope.