

Homework 6

Due midnight, Thursday, 3/5/2015

Please submit your homework solutions online at <http://www.dci.pomona.edu/tools-bin/cs081upload.php>. If you have more than one file to be turned in, please put it in a folder and zip it up before turning it in.

Problems from the texts are given in the form c.n where c is the chapter and n is the problem number. Thus problem 2.7 is problem 7 from Chapter 2.

1. (15 points) **Classical Rules**

Show that the following rules of inference are all equivalent to one another, in the sense that adding one to the constructive rules gives a logical system that proves the same theorems as adding any other to the constructive rules.

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| <p>(a) $\frac{\neg\neg\phi}{\phi} \neg\neg e$</p> <p>(b) $\frac{}{\neg\phi \vee \phi} \text{LEM}$</p> | <p>(c) $\frac{\phi \rightarrow \psi \quad \neg\phi \rightarrow \psi}{\psi} \text{Cases}$</p> <p>(d) $\frac{\phi \rightarrow \psi}{\neg\phi \vee \psi} \text{CDI}$</p> <p>(e) PBC</p> |
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Suggestion: For two rules, X and Y, show that X can be simulated by the constructive rules and Y. This will show that any theorem provable with X is also provable with Y. Create a cycle of five such proofs that run through all the rules. You may choose any order for the cycle, as long as there is a loop with all five rules.

LEM, which stands for “law of the excluded middle,” is standard jargon. PBC is used by Huth and Ryan for “proof by contradiction.” CDI stands for “classical definition of implication”; it is made up especially for this problem.

2. (10 points) **De Morgan laws**

In class, we gave a constructive proof of one direction of a De Morgan law, $\phi \vee \psi \vdash \neg(\neg\phi \wedge \neg\psi)$. Is the converse, $\neg(\neg\phi \wedge \neg\psi) \vdash \phi \vee \psi$, constructively provable? If so, give a constructive proof. If not, give a classical proof and show how to derive the classical contradiction rule (or an equivalent rule from Problem 1) from it.

3. (10 points) **Adequacy**

Problem 1.5.3ab from H & R page 87. Problem 1.5.3c can be done for 5 points extra credit.

4. (5 points) **Conjunctive Normal Form**

Problem 1.5.7b from H & R page 88.

5. (5 points) **Horn Clauses**

Problem 1.5.16 from H & R page 90.