# Homework 5

Due midnight, Thursday, 2/26/2015

Please submit your homework solutions online at `http://www.dci.pomona.edu/tools-bin/cs081upload.php`. If you have more than one file to be turned in, please put it in a folder and zip it up before turning it in.

Problems from the texts are given in the form c.n where c is the chapter and n is the problem number. Thus problem 2.7 is problem 7 from Chapter 2.

1. (15 points) **CFL's over alphabet of size 1**

   (a) Let $L$ be a context-free language over the one-element alphabet $\{a\}$. Show that $L$ is regular. Hint: Let $p$ be the constant from the Pumping Lemma. Show that for each $x \in L$ with $p \leq |x|$, there are natural numbers $r$ and $s$,with $s$ no greater than $p$, such that the set $A_{r,s} = \{a^{r+is} | 0 \leq i\}$ is a subset of $L$. Then show that $L$ is the union of a (possibly empty) finite set of strings of length less than $p$ together with finitely many sets of the form $A_{r,s}$ (and note that $A_{r,s}$ is regular).

   (b) Prove that the set $\{a^n | n \text{ is a prime number}\}$ is not context-free. *Hint: Use the first part to make your life simpler.*

2. (5 points) **Closure**

   Rich 13.19. Hint: Use L = $\{1a^nb^{2n} \cup 2a^nb^n | n \geq 0\}$

3. (5 points) **Algorithms**

   Rich 14.1d.

4. (10 points) **Parsing with CKY**

   Consider the Chomsky Normal Form grammar $S \to AB$, $A \to a$, and $B \to AB \mid b$. Use the CKY algorithm to fill in a table like the one in class for each of the strings $aab$ and $aba$. Use these tables to determine if each of the strings is in the language generated by the grammar. If they are, explain how to use the table to produce a derivation.

5. (20 points) **Unique Readability**

   For this problem consider formulas of propositional logic to be exactly those generated by the grammar given in class (and thus only those formulas that are fully parenthesized.) Consider the following function on symbols from the alphabet $\mathcal{A}$ of propositional logic.

   $K(() = -2$
   $K()) = 2$
   $K(p) = 1$, for a proposition letter $p$
   $K(\top) = K(\bot) = 1$
   $K(\neg) = 0$
   $K(\wedge) = K(\vee) = K(\to) = -1$

   We extend the function to strings of symbols, $K^* : \mathcal{A}^* \to N$ in the natural way: $K^*(\epsilon) = 0$ and $K^*(as) = K(a) + K^*(s)$. You may use without proof the fact that $K^*(st) = K^*(s) + K^*(t)$.

   (a) Prove by induction that if $\phi$ is a formula of propositional logic, then $K^*(\phi) = 1$.

(b) A string $s$ is a *proper prefix* of a formula $u$ if there is a non-empty string $t$ such that $st = u$. Prove that $K^*(s) < 1$ for any string $s$ which is a proper prefix of some formula.

(c) Conclude that a formula cannot be a proper prefix of another formula. Informally explain why this fact tells us that there is only one way to parse a formula. The latter property is called "unique readability."

(d) Give an example of a strings $s$ and $u$ such that $s$ is a proper prefix of $u$ and $K^*(s) = K^*(u) = 1$ Why does your example not contradict the conclusions above?