Homework 3

Due midnight, Thursday, 2/12/2015

Happy (almost) Valentine's Day!

Please submit your homework solutions online at `http://www.dci.pomona.edu/tools-bin/cs081upload.php`. If you have more than one file to be turned in, please put it in a folder and zip it up before turning it in.

Problems from the texts are given in the form c.n where c is the chapter and n is the problem number. Thus problem 2.7 is problem 7 from Chapter 2.

1. (20 points) **Minimization algorithm**

   In this problem you will be creating an algorithm for minimizing finite state machines. Let $M = \{K, \Sigma, \delta, s, A\}$ and for each $q \in K$, let $M_q = \{K, \Sigma, \delta, q, A\}$ (that is, a machine that differs from M only by changing the start state). For $k \geq 0$, say states p and q of M are *k-distinguishable* if there is some input string of length at most k which is accepted by one of M_p and M_q, but not the other. Otherwise they are *k-indistinguishable*. Two states are equivalent if they are k-indistinguishable for every k.

   (a) Show that if a deterministic finite state machine has two equivalent states, one can be eliminated; but if all pairs of distinct states are inequivalent, then the machine is as small as possible.

   (b) Show that states p and q are k-indistinguishable if and only if

      i. both are accepting or both are non-accepting, and

      ii. if $k > 0$, then for each $a \in \Sigma$, $\delta(p, a)$ and $\delta(q, a)$ are $(k-1)$-indistinguishable.

   (c) Define a series of equivalence relations $\equiv_0, \equiv_1, \ldots$ on the states of M as follows:

      - $p \equiv_0 q$ if and only if p and q are accepting or both are non-accepting.
      - $p \equiv_{k+1} q$ if and only if $p \equiv_k q$ and, for each $a \in \Sigma$, $\delta(p, a) \equiv_k \delta(q, a)$.

      Show that $p \equiv_k q$ if and only if p and q are k-indistinguishable.

   (d) Show that there is an $n \leq |K|$ such that $\equiv_n$ and $\equiv_{n+1}$ are identical. Then by the previous part and the definition of equivalent states, all equivalent states can be discovered by carrying through the inductive definition of $\equiv_{k+1}$ at most $|K|$ times. Use part a of this problem to describe an algorithm to minimize a given deterministic finite state machine.

2. (20 points) **Pumping Lemma, etc.**

   Problem 8.1n, s

3. (5 points) **Regular Languages**

   Let $G = (V, \Sigma, R, S)$ where $V = \{a, b, S\}$, $\Sigma = \{a, b\}$, and R contains:

   - $S \to aSb$
   - $S \to aSa$
   - $S \to bSa$
   - $S \to bSb$

- $S \to \epsilon$.

Show $L(G)$ is regular.

4. (15 points) **Context-free Language**

   Let L $= \{w \in \{a,b\}^* | w = w^{rev}\}$.

   (a) Design a cfg generating L.

   (b) Design a pda accepting L.

   (c) Design a cfg generating the complement of L