Homework 2

Due 11:59 p.m., Thursday, 2/5/2015

Please submit your homework solutions online at `http://www.dci.pomona.edu/tools-bin/cs081upload.php`. If you have more than one file to be turned in, please put it in a folder and zip it up before turning it in.

Problems from the texts are given in the form c.n where c is the chapter and n is the problem number. Thus problem 2.7 is problem 7 from Chapter 2.

1. (10 points) **Closure**

   Problem 8.7cd from Rich. You may use any of the representations of regular languages discussed in class. Do provide an argument as to why your construction gives all the strings of the desired language and no extras.

2. (10 points) **More Closure**

   In this problem, I want you to show the reverse of problem 8.7d.

   Let Σ, Δ be alphabets and $h : \Sigma \to \Delta^*$. We can extend h to Σ^* as follows:

   $h(\epsilon) = \epsilon$
   $h(wa) = h(w)h(a)$ for any $w \in \Sigma^*, a \in \Sigma$

   Any function $h : \Sigma^* \to \Delta^*$ defined in this way from a function $h : \Sigma \to \Delta$ is called a homomorphism.

   Let $h : \Sigma^* \to \Delta^*$ be a homomorphism. Show that if $L \subset \Delta^*$ is regular (i.e., accepted by a finite state machine) then so is $\{w \in \Sigma^* | h(w) \in L\}$.

   Hint: Start from a deterministic finite state machine M accepting L and construct one with the same states which, when it reads an input symbol a delivers $h(a)$ *in its imagination* to M for processing.

3. (10 points) **Regular Languages**

   Let $\Sigma_3 = \left\{ \begin{bmatrix}0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}, \ldots, \begin{bmatrix}1\\1\\1\end{bmatrix} \right\}$.

   The alphabet Σ_3 has eight letters, consisting of all the size-three columns of 0's and 1's. Consider each row to be a binary number, and let

   $$B = \{w \mid \text{the bottom row is the sum of the two upper rows}\}.$$

   For example, $\begin{bmatrix}0\\0\\1\end{bmatrix}\begin{bmatrix}1\\0\\0\end{bmatrix}\begin{bmatrix}1\\1\\0\end{bmatrix} \in B$, but $\begin{bmatrix}0\\0\\1\end{bmatrix}\begin{bmatrix}1\\0\\1\end{bmatrix} \notin B$, Show that B is regular.1 Hint: Working with $B^{\mathcal{R}}$ is easier. You may use the result of problem 1.

4. (10 points) **Regular expressions**

   Let $\Sigma = \{a, b\}$, and let A be the set of words w over Σ for which w contains an even number of a's or an odd number of b's. Give a regular expression for the language A.

5. (5 points) **Minimizing FSMs**

   Problem 5.12 from Rich