

Homework 4

Due midnight, Thursday, 2/21/2013

Please submit your homework solutions online at <http://www.dci.pomona.edu/tools-bin/cs081upload.php>. If you have more than one file to be turned in, please put it in a folder and zip it up before turning it in.

Problems from the texts are given in the form c.n where c is the chapter and n is the problem number. Thus problem 2.7 is problem 7 from Chapter 2.

1. (5 points) **Non-context-free Languages**

Rich 13.1.i. *Hint: You will need to look at cases. See the examples in the book.*

2. (15 points) **CFL's over alphabet of size 1**

(a) Let L be a context-free language over the one-element alphabet $\{a\}$. Show that L is regular.

Hint: Let p be the constant from the Pumping Lemma. Show that for each $x \in L$ with $p \leq |x|$, there are natural numbers r and s , with s no greater than p , such that the set $A_{r,s} = \{a^{r+is} \mid 0 \leq i\}$ is a subset of L . Then show that L is the union of a (possibly empty) finite set of strings of length less than p together with finitely many sets of the form $A_{r,s}$ (and note that $A_{r,s}$ is regular).

(b) Prove that the set $\{a^n \mid n \text{ is a prime number}\}$ is not context-free. *Hint: Use the first part to make your life simpler.*

3. (10 points) **Closure**

Rich 13.13.ef.

4. (5 points) **Closure**

Rich 13.19. *Hint:* Use $L = \{1a^n b^{2n} \cup 2a^n b^n \mid n \geq 0\}$

5. (15 points) **Closure**

Rich 13.22bcd. *Be careful, these are tricky!*

6. (5 points) **Algorithms**

Rich 14.1d.

7. (10 points) **Parsing with CKY**

Consider the Chomsky Normal Form grammar $S \rightarrow AB$, $A \rightarrow a$, and $B \rightarrow AB \mid b$. Use the CKY algorithm to fill in a table like the one in class for each of the strings aab and aba . Use these tables to determine if each of the strings is in the language generated by the grammar. If they are, explain how to use the table to produce a derivation.