Homework 10

Due midnight, Thursday, 4/25/2013

Please submit your homework solutions online at `http://www.dci.pomona.edu/tools-bin/cs081upload.php`. If you have more than one file to be turned in, please put it in a folder and zip it up before turning it in.

1. (10 points) **Decidability**

   Recall that L^* is defined as $\{w_1 \dots w_n \mid \forall 1 \leq i \leq n, w_i \in L\}$

   (a) Assume L is a decidable language. Show that L^* is also decidable.

   (b) Assume L is a semi-decidable language. Show that L^* is also semi-decidable.

2. (5 points) **Trick question!**

   Is there a Turing machine M satisfying the following condition?

   > The machine M halts on each input word w, and it accepts the word w if and only if there is a proof of $P = NP$.

3. (10 points) **Decidability**

   A language L is *co-semi-decidable* if its complement $\overline{L}$ is semi-decidable. Let A and B be disjoint co-semi-decidable languages. Show that there is a decidable language S such that $A \subseteq S$ and $B \subseteq \overline{S}$. Hint: Think about the value of $\overline{A} \cup \overline{B}$ and how you can use that to get a decidable language satisfying the given constraints.

4. (10 points) **Reduction**

   Prove that the following question is undecidable. Given a Turing machine M, is $L(M)$ finite? (Do not use Rice's Theorem in your proof.)

5. (10 points) **Reduction**

   One of the following sets is semi-decidable, and the other is not. Which is which? Give proofs for both.

   (a) $\{M \mid L(M)$ contains at least 481 elements$\}$

   (b) $\{M \mid L(M)$ contains at most 480 elements$\}$

   Hint: Recall that a set is decidable if and only if both it and its complement are semi-decidable. Use the contrapositive to show one of these is not semi-decidable. (Do not use Rice's Theorem in your proof.)