Homework 6

Due midnight, Thursday, 3/1/2012

Please submit your homework solutions online at `http://www.dci.pomona.edu/tools-bin/cs081upload.php`. If you have more than one file to be turned in, please put it in a folder and zip it up before turning it in.

Problems from the texts are given in the form c.n where c is the chapter and n is the problem number. Thus problem 2.7 is problem 7 from Chapter 2.

1. (10 points)**Adequacy**

   Problem 1.5.3ab from H & R page 87. Problem 1.5.3c can be done for 5 points extra credit.

2. (5 points) **Conjunctive Normal Form**

   Problem 1.5.7b from H & R page 88.

3. (5 points) **Horn Clauses**

   Problem 1.5.16 from H & R page 90.

4. (5 points) **Predicate Logic**

   Problem 2.1.3 from H & R page 157.

5. (10 points) **Free & Bound Variables**

   (a) In the formula below, or a copy of it, underline the occurrences of free variables and circle the occurrences of bound variables. (Consider the variable right after a quantifier to be bound.)

   $$\forall z \left(P(x) \wedge \forall x \, \exists y \, \big(Q(y, f(z)) \rightarrow R(g(g(w, x), f(y)))\big) \right)$$

   (b) Let ϕ be the formula in part a. Write out the formulas below, renaming bound variables when necessary to avoid "capture."

      i. $\phi[f(y)/x]$
      ii. $\phi[f(y)/z]$
      iii. $\phi[f(y)/w]$

6. (10 points) **Proofs**

   Exactly one of the inferences below is correct. Give a (constructive) proof of the correct member and a short intuitive explanation of why the other is incorrect.

   (a) $\forall x \, (\phi \vee \psi) \vdash \forall x \, \phi \vee \forall x \, \psi$ and
   (b) $\forall x \, \phi \vee \forall x \, \psi \vdash \forall x \, (\phi \vee \psi)$

7. (10 points) **Bad Proof**

   Recall the axioms for an equivalence relation $\sim$:

   A0. $\forall x \, \big(x \sim x\big)$
   A1. $\forall x \, \forall y \, \big(x \sim y \rightarrow y \sim x\big)$

A2. $\forall x\,\forall y\,\forall z\,\big(x \sim y \rightarrow \big(y \sim z \rightarrow x \sim z\big)\big)$

Consider the following proof sketch that purports to prove that $\mathrm{A1}, \mathrm{A2} \vdash \mathrm{A0}$. Formalize the sketch and use your formulation to identify the error in it.

> Let x be arbitrary and suppose that $x \sim y$. Then by A1, $y \sim x$. Using these two facts, we may use A2 to conclude that $x \sim x$. Because x was arbitrary, we have $\forall x\,\big(x \sim x\big)$.

8. (10 points) **Proofs**

   Problem 2.3.11ac from H & R page 162.