Homework 12

Due midnight, Thursday, 4/26/2012

Please submit your homework solutions online at `http://www.dci.pomona.edu/tools-bin/cs081upload.php`. If you have more than one file to be turned in, please put it in a folder and zip it up before turning it in.

1. (10 points) **Computable Functions and semi-decidability**

   (a) Show that a set L is semi-decidable iff it is the domain of a Turing-computable (partial) function.

   Hint: ($\Rightarrow$) Use the TM accepting L to define a function whose domain is L. Note that it doesn't matter what the function returns as a value. Reverse the hint for the other direction.

   (b) Show that a set L is semi-decidable iff it is the range of a Turing-computable (partial) function.

   Hint: ($\Rightarrow$) Use the dovetailing technique to compute a function. If you try hard enough you can actually show (for two points extra credit) that a semi-decidable set L is either finite or the range of an injective total Turing computable function. For another point of extra credit show a semi-decidable set L is either empty or the range of a total Turing computable function.

2. (20 points) **More Diagonalization**

   Let $TOTAL_{\text{TM}} = \{\langle M \rangle \mid M$ is a Turing machine that halts on *all* inputs$\}$. In class we showed that $TOTAL_{TM}$ is not decidable. Prove that neither $TOTAL_{\text{TM}}$ nor its complement is recognizable.

   Hints: Recall that a language is decidable if and only if both it and its complement are recognizable.

   For $TOTAL_{\text{TM}}$, assume that it is the range of a computable function (see the previous problem) and then diagonalize to obtain a contradiction. Then show that if the complement of $TOTAL_{\text{TM}}$ is recognizable, then so is the complement of H_{TM}.

3. (10 points) **Programming language undecidability**

   Problem 21.7c in Rich, page 484

4. (10 points) **CFG undecidability**

   Problem 22.7c in Rich, page 509.