Lecture 19: Binary Search & Splay Trees

CS 62
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Exam Monday

- In class: 50 minutes
- Sample Exams on-line
- Covers everything through Splay trees
- Studying essential
  - Do form study groups
  - Do problems from sample exams
  - Do problems from text

Assignment

- Work first on World & Species classes
  - Need JUnit for both and turn in World
  - Each creature keeps a reference to its species so it can follow the program.
  - Moves hop, left, right, and infect use up turn
    - if's and goto are free

BST

- A binary tree is a binary search tree iff
  - it is empty or
  - if the value of every node is both greater than or equal to every value in its left subtree and less than or equal to every value in its right subtree.

Implementation

- Focus on trickiest methods:
  - add, get, & remove
  - protected methods: locate, predecessor, and removeTop

```
// @pre root and value are non-null
// @return: 1 - existing tree node with the desired value, or
//          2 - the node to which value should be added
   protected BinaryTree<E> locate(BinaryTree<E> root, E value){
       E rootValue = root.value();
       BinaryTree<E> child;
       if (rootValue.equals(value)) return root; // found at root
       // look left if less-than, right if greater-than
       if (ordering.compare(rootValue,value) < 0) {
           child = root.right();
       } else {
           child = root.left();
       }
       // no child there: not in tree, return this node,
       // else keep searching
       if (child.isEmpty()) {
           return root;
       } else {
           return locate(child, value);
       }
   }
```

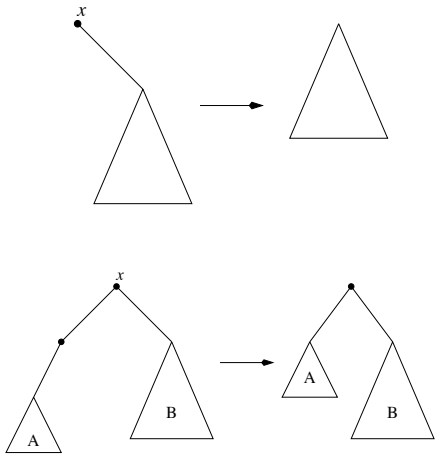
```
protected BinaryTree<E> predecessor(BinaryTree<E> root) {
     BinaryTree<E> result = root.left();
     while (!result.right().isEmpty()) {
         result = result.right();
     }
     return result;
}

protected BinaryTree<E> successor(BinaryTree<E> root) {
     BinaryTree<E> result = root.right();
     while (!result.left().isEmpty()) {
         result = result.left();
     }
     return result;
}
```

```
  public void add(E value) {
       BinaryTree<E> newNode = new BinaryTree<E>(value,EMPTY,EMPTY);
       // add value to binary search tree
       // if there's no root, create value at root
       if (root.isEmpty()) {
           root = newNode;
       } else {
           BinaryTree<E> insertLocation = locate(root,value);
           E nodeValue = insertLocation.value();
           // The location returned is the successor or predecessor
           // of the to-be-inserted value
           if (ordering.compare(nodeValue,value) < 0) {
               insertLocation.setRight(newNode);
           } else {
               if (!insertLocation.left().isEmpty()) {
                   // if value is in tree, we insert just before
                   predecessor(insertLocation).setRight(newNode);
               } else {
                   insertLocation.setLeft(newNode);
               }
           }
       }
       count++;
   }
```

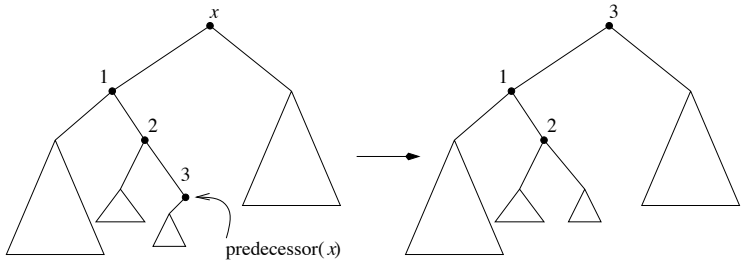
Remove node

- Remove topmost node.
- Easy cases:
  - no left subtree, or no right subtree -- easy, they are new tree
  - left child has no right subtree



General Case

- Left Child has a right subtree:



Remove method

- Locate element to be deleted
- RemoveTop of node rooted at element
- Hook up resulting tree as child of elt's parent.
- O(h), where h is height of tree.
  - O(h) to find,
  - Could be another O(h) to find predecessor
  - Constant to patch back together.

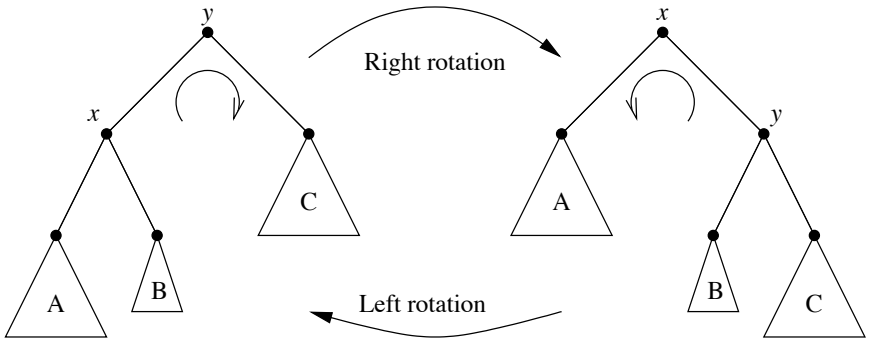
Complexity

- Add, get, contains, remove
  - Proportional to height of tree
- Can we guarantee O(log n)?
  - Only if we can keep them balanced!!
  - Special binary search trees that stay balanced:
    - AVL trees
    - Red-black trees
  - We'll do splay tree, which doesn't guarantee balance
    - but guarantees good average behavior
    - easier to understand than alternatives
    - better than others if likely to go back to recent nodes

Splay Trees

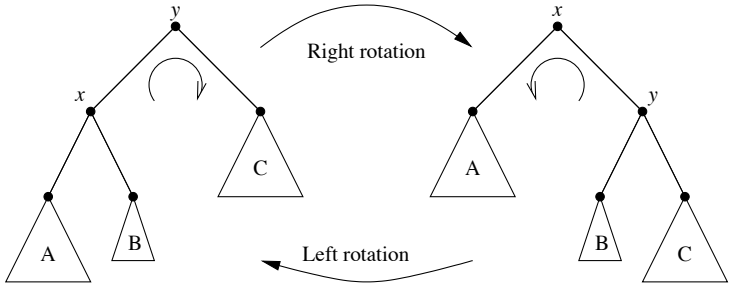
Rotating Trees

- Key idea: Rotate node higher in tree while keeping it in order.



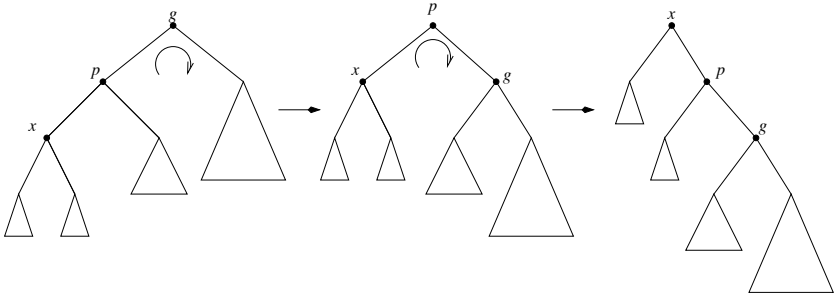
Rotating Trees

- Rotate x to root, while maintain BST structure
  - All nodes in subtree A go up one level, all in C go down one level, all in B stay same.
  - See code in BinaryTree



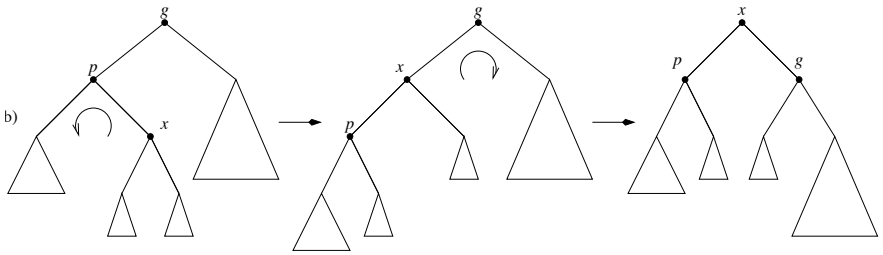
Shifting elements toward root

- Move x up two levels w/ two rotations
- If x is left child of a left child ...



Shifting elements toward root

- If x is a right child of a left child.



Symmetric if interchange left and right

Splay Tree

- Idea behind splay tree.
  - Every time find, get, add: or remove an element x, move it to the root by a series of rotations.
  - Other elements rotate out of way while maintaining order.
- Splay means to spread outwards