Lecture 18: Heaps & Heapsort

CS 62
Spring 2015
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Exam Next Monday

- In class: 50 minutes
- Covers everything through Splay trees
- Studying essential
  - Form study groups
  - Do problems from sample exams (*soon* on web page)
  - Do problems from text

Exam Topics

- Pre and post-conditions
- ArrayLists
- Java Graphics/GUI
- Analysis of Algorithms: Big-O
- Induction/Sorting
- Iterators
- Linked Lists
- Stacks
- Queues
- Trees
- Binary Search Trees/ Splay trees

Lab

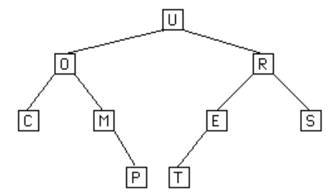
- Iterators and closures

Next Project: Darwin

- Darwin: Program creatures w/zombie-like behavior!
- Final version due in 1 1/2 weeks
  - Part 1 due Friday.
  - Contest

Array Representation

- data[0..n-1] can hold values in trees
  - left subtree of node i in 2*i+1, right in 2*i+2,
  - parent in (i-1)/2



Indices:	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
data[]:	U	O	R	C	M	E	S	-	-	-	P	T	-	-	-

Min-Heap

- Min-Heap H is complete binary tree s.t.
  - H is empty, or
  - Both of the following hold:
    - The value in root position is smallest value in H
    - The left and right subtrees of H are also heaps.
      Equivalent to saying parent ≤ both left and right children
- Excellent implementation for priority queue
  - Dequeue elements w/lowest priority values before higher

PriorityQueue

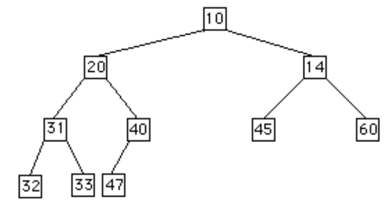
```
public interface PriorityQueue<E extends Comparable<E>>
{

    /**
     * @pre !isEmpty()
     * @return The minimum value in the queue.
     */
    public E remove();
    public E getFirst();
    public void add(E value);
    public boolean isEmpty();
    public int size();
    public void clear();
}
```

Implementations

- As regular queue (array or linked) where either keep in order or search for lowest to remove:
  - One of add or remove will be O(n)
- Heap representation (in arraylist) is more efficient: O(log n) for both add and remove.
  - Insert into heap:
    - Place in next free position,
    - “Percolate” it up.
  - Delete:
    - remove root,
    - move smallest child up to fill gaps, repeat

Insert 15:



IndexRange:	0	1	2	3	4	5	6	7	8	9	10
data:	10	20	14	31	40	45	60	32	33	47	-

IndexRange:	0	1	2	3	4	5	6	7	8	9	10
data:	10	20	14	31	*40*	45	60	32	33	47	15

IndexRange:	0	1	2	3	4	5	6	7	8	9	10
data:	10	*20*	14	31	15	45	60	32	33	47	40

IndexRange:	0	1	2	3	4	5	6	7	8	9	10
data:	10	15	14	31	20	45	60	32	33	47	40

See VectorHeap code

Sorting with Trees

Tree Sort

- Build Binary search tree (later)
- Do Inorder traversal, adding elts to array
  - Inorder traversal: O(n)
  - Building tree:
    - log 1 + log 2 + ... + log n = O(n log n) in best (& average) case
    - $O(n^2)$ in worst case
- O(n log n) in best & average case
- $O(n^2)$ in worst case :-(*What is worst case?*
- Heapsort is always better!

Heapsort

- Make vector into a heap:
  - n add operations = O(n log n)
- Remove elements in order
  - n remove operations = O(n log n)
- Total: O(n log n)
  - If clever can make into heap in O(n)
  - ... but still O(n log n) total.

Comparing Sorts

- Quicksort: fastest on average O(n log n), but worst case is $O(n^2)$ & takes O(log n) extra space
- Heapsort: O(n log n) in average & worst case. No extra space.
  - Bit slower on average than quick & mergesorts.
- Mergesort: O(n log n) in average and worst case. O(n) extra space.
  - Performs well on external files where not all fit in memory.

Binary Search Trees

BST

- A binary tree is a binary search tree iff
  - it is empty or
  - if the value of every node is both greater than or equal to every value in its left subtree and less than or equal to every value in its right subtree.

Implementation

- Focus on trickiest methods:
  - add, get, & remove
  - protected methods: locate, predecessor, and removeTop

```
// @pre root and value are non-null
// @post returned: 1 - existing tree node with the desired value, or
//          2 - the node to which value should be added
   protected BinaryTree<E> locate(BinaryTree<E> root, E value){
       E rootValue = root.value();
       BinaryTree<E> child;
       if (rootValue.equals(value)) return root; // found at root
       // look left if less-than, right if greater-than
       if (ordering.compare(rootValue,value) < 0) {
           child = root.right();
       } else {
           child = root.left();
       }
       // no child there: not in tree, return this node,
       // else keep searching
       if (child.isEmpty()) {
           return root;
       } else {
           return locate(child, value);
       }
  }
```