Lecture 40: Minimum Spanning Trees

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Spanning Trees

- A spanning tree T of a graph G is a subset of the edges of G such that:
  - T contains no cycles and
  - Every vertex in G is connected to every other vertex using just the edges in T.
- An unconnected graph has no spanning trees.
- A connected graph G will have at least one spanning tree; it may have many.

Minimum Spanning Trees

- A weighted graph is a graph that has a weight associated with each edge.
- If G is a weighted graph, the cost of a tree is the sum of the costs (weights) of its edges.
- A tree T is a minimum spanning tree of G iff:
  - it is a spanning tree and
  - there is no other spanning tree whose cost is lower than that of T.

Don't care about the root!

Minimum Spanning Trees

- Can we find an MST without searching all the possible trees?
Minimum Spanning Trees

• Application:
  • The cheapest way to lay cable that connects a set of points is along a minimum spanning tree that connects those points.
• Many algorithms exist to find minimum spanning trees, most run in $O(e \log e)$ time.
• In 1995 Karger, Klein & Tarjan found a linear time randomized algorithm, but there is no known linear time deterministic algorithm

Kruskal’s Algorithm

• Create forest $F$ with no edges, using vertices in $V$
  • Sort the edges in the graph by their weight (smallest to largest)
  • For each edge $e$ in sorted order:
    • if $e$ connects two different trees in $F$, then add $e$ to $F$

Side-Remarks

• What is the size of a spanning tree of $G$ if $G$ has $n$ vertices?
• Why must minimum cost spanning tree include least weight edge in graph?

Sample Graph

- $(1,2):1$
- $(2,3):2$
- $(4,5):3$
- $(6,7):3$
- $(1,4):4$
- $(2,5):4$
- $(4,7):4$
- $(3,5):5$
- $(2,4):6$
- $(3,6):6$
- $(5,7):7$
- $(5,6):8$
Graph Algorithms

- Very important in practice!
- Sophisticated data structures
- Careful analysis of
  - correctness
  - complexity
- CS 140: Algorithms