Lecture 19: Balanced Binary Trees

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Keeping Trees Trimmed

- Lots of operations are $O(h)$
- But our guarantee is only: $\log n \leq h \leq n$
- Can we do better?

Keeping Trees Trimmed

- We can rotate a left child upwards:
  1. Give our right subtree to our parent as a left subtree
  2. Set our parent as our own right subtree
  3. Take our parent's old position
- All of our left descendants move up
- All of our parent's right descendants move down
- Our right descendants don't change height

Keeping Trees Trimmed

- The symmetric operation can rotate to the right
- A sequence of rotations can move a node to the root
- Bystander nodes end up more-balanced

Splay Trees

- Every time we call `add`, `contains`, or `remove`, rotate up to the root
  - Side-effects of rotation give average-case O(log n) tree height
  - Worst case is still O(n)
  - But all O(h) operations are now average-case O(log n)

Demo

Splay Tree Demo

Sketch

```
void splay(BinaryTree<E> target) {
  while (this.root != target) {
    this.rotate(target.parent, target);
  }
}
```

Fixing Sticks

- Simple "rotate-up" strategy doesn't fix sticks
- Splay operations:
  - Zig
  - Zig-zig
  - Zig-zag

Splay Operations

- Zig: Rotate *self* once L/R
  (when you have no grandparent)
- Zig-zig: Rotate *parent*, then *self*
  (when you're L/L or R/R)
- Zig-zag: Rotate *self*, then *self*
  (when you're L/R or R/L)

Height Changes

- Zig:
  - One of our subtrees → -1
  - Our other subtree → no change
  - Parent's other subtree → +1

Height Changes

- Zig-zig:
  - Grandparent's other → +1
  - Parent's other → no change
  - Both of ours → -1
  - …plus…
  - Grandparent's other → +1
  - Parent's other → +1
  - One of ours → -1
  - Our other → no change

Height Changes

- Zig-zig:
  - Grandparent's other → +2
  - Parent's other → +1
  - One of ours → -2
  - Our other → -1

Height Changes

- Zig-zag:
  - Grandparent's other → +1
  - Parent's other → +0
  - Both of ours → -1

Changes for Sticks

- Our children: lots
- Parent/grandparent other children: zero

Average change: shorter

Changes for Balanced Nodes

- Our children: lots
- Parent/grandparent other children: same

Average change: none