Computability and Logic Homework 9

Due: Thursday, November 17, 2005

1. Recall that a formula A is ***valid*** if $\models A$; we say A is ***satisfiable*** if there exists some truth assignment η such that $\mathcal{T}_\eta(A) = \texttt{tt}$. For each of the following, prove or give a counterexample as appropriate:

a. If $A \supset B$ is valid and A is valid, then B is valid.

b. If $A \supset B$ is satisfiable and A is satisfiable, then B is satisfiable.

c. If $A \vee B$ is satisfiable, then A is satisfiable or B is satisfiable.

d. If $A \wedge B$ is valid, then A is valid and B is valid.

2. Give derivations for each of the following judgments. You may present your derivations in either list form or tree form as described in the logic handout.

a. $(A \supset B), (B \supset C) \vdash A \supset C$

b. $(A \vee B), (\neg B \vee C) \vdash A \vee C$

c. $\vdash (A \vee (A \wedge B)) \supset (A \wedge (A \vee B))$

3. Negation Give deductions for each of the following. (Or, prove that deductions exist using deductions you've shown to exist earlier together with structural properties and/or Cut and/or inference rules.)

a. $A \vdash \neg\neg A$

b. $\neg\neg\neg A \vdash \neg A$ (*without* using the RAA rule.)

c. $\neg\neg\neg A \vdash \neg A$ (*using* the RAA rule to produce a shorter deduction than in part (b).)

d. $\vdash \neg\neg(A \vee \neg A)$

4. Consider this alternative formulation of the $\neg I$ rule:

$$\frac{\Gamma, A \vdash \neg A}{\Gamma \vdash \neg A}\ (\neg I')$$

In proving the following, do not appeal to the completeness theorem. Prove that the requested derivations exist by showing how to construct them directly.

a. Show that for any Γ and C, if there is a derivation of $\Gamma \vdash C$ that uses this new rule, there is a derivation of the same judgment that uses the original rule instead.

b. Show that for any Γ and C, if there is a derivation of $\Gamma \vdash C$ that uses the original $\neg I$ rule, there is a derivation of the same judgment that uses the $\neg I'$ rule instead.

5. De Morgan's Laws Give derivations:

a. $A \vee B \vdash \neg(\neg A \wedge \neg B)$

b. $\neg(\neg A \vee \neg B) \vdash A \wedge B$

c. $\neg(A \wedge \neg B) \vdash A \supset B$