Computability and Logic Homework 5

Due: Thursday, October 6, 2005

Problems 1 and 4 are worth 10 points each; problems 2 and 3 are worth 15 points each.

1. CFG's into PDA's Consider the following context-free grammar:

$$S \to \Lambda \mid aSbS \mid bSaS$$

a. What is the language generated by this grammar? (You need not prove your answer.)

b. Use the construction given in class to find a pushdown automaton accepting the language generated by this grammar.

c. Show the steps your automaton takes to accept the string $aabbba$.

2. Pushdown Automaton Execution Let $M = (Q, V, s, R, T)$ be a pushdown automaton over Σ. Using induction on the number of steps in the computation, prove that if $(p, w, \gamma) \mapsto^* (q, w', \gamma')$ then for any $v \in \Sigma^*$ and any $\delta \in V^*$, $(p, wv, \gamma\delta) \mapsto^* (q, w'v, \gamma'\delta)$.

3. Simple Pushdown Automata Recall from lecture that a pushdown automaton $M = (Q, V, s, R, T)$ is *simple* if for every transition $p \xrightarrow{a,\alpha/\beta} q \in T$:

1. $q \neq s$ (that is, the automaton never returns to its start state).

2. If $p = s$ then $a = \Lambda$, $\alpha = \Lambda$ and $|\beta| = 1$ (that is, the first step in any computation consumes no input and pushes a single symbol onto the stack).

3. If $p \neq s$ then $|\alpha| = 1$ and $|\beta| \leq 2$ (that is, every step after the first step pops exactly one symbol from the stack and replaces it with at most two others).

Suppose M is a simple pushdown automaton. Prove that if $(p, w, \gamma_1\gamma_2) \mapsto^* (q, \Lambda, \Lambda)$ and $p \neq s$, then there must exist a state r and strings w_1 and w_2 such that $w = w_1 w_2$ and $(p, w_1, \gamma_1) \mapsto^* (r, \Lambda, \Lambda)$ and $(r, w_2, \gamma_2) \mapsto^* (q, \Lambda, \Lambda)$. Your proof should be by induction on the number of steps in the given computation.

4. PDA's into CFG's Consider the pushdown automaton having states $\{s, t, u, v\}$, (s is the start state, v is the only final state), stack alphabet $\{a, \#\}$ and transitions:

$$s \xrightarrow{\Lambda,\Lambda/\#} t$$
$$t \xrightarrow{a,\#/a\#} t \qquad t \xrightarrow{a,a/aa} t$$
$$t \xrightarrow{\Lambda,\#/\#} u \qquad t \xrightarrow{\Lambda,a/a} u$$
$$u \xrightarrow{b,a/\Lambda} u$$
$$u \xrightarrow{\Lambda,\#/\Lambda} v$$

a. What is the language accepted by this automaton? Briefly and informally describe its operation.

b. Verify for yourself that this pushdown automaton is simple; then, use the construction given in class to find a context-free grammar that generates the language it accepts. This grammar will contain many rules that cannot be used to derive strings of terminals; you do not need to show these rules. (**Note:** You already know an elegant grammar for this language, but it is not a correct answer to this problem!)

c. Show a derivation of $aabb$ using the grammar you produced in part (b).