

Computability and Logic Homework 4

Due: Thursday, September 29, 2005

Problems 1 and 2 are worth 15 points each. Problems 3 and 4 are worth 10 points each.

1. Use the pumping lemma (carefully!) to prove that $\{a^n b a^m b a^{m+n} \mid m, n \geq 1\}$ is not regular.

2. **Multiplication** Recall from Homework 2 the alphabet Σ consisting of all 3-place binary vectors:

$$\Sigma = \left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \dots, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$$

In Homework 2 you showed that the set of sequences of column vectors in Σ that represent correct binary additions is regular. We can also use strings over this alphabet to represent binary **multiplications**. An example of a correct multiplication is

$$\begin{array}{r} 0 \ 0 \ 1 \ 0 \ 1 \ 0 \\ \times 0 \ 0 \ 0 \ 1 \ 0 \ 1 \\ \hline 1 \ 1 \ 0 \ 0 \ 1 \ 0 \end{array} \quad \text{represented by} \quad \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

Use the pumping lemma to show that the set of strings over Σ^* representing correct binary multiplications is **not** regular. (Hint: Consider the multiplication $(2^n + 1)(2^n + 1)$.)

3. **Derivations** Consider the following grammar over $\Sigma = \{a, b\}$:

$$\begin{aligned} S &\rightarrow AA \\ A &\rightarrow AAA \\ A &\rightarrow a \\ A &\rightarrow bA \\ A &\rightarrow Ab \end{aligned}$$

- Give at least four distinct derivations of the string $babbab$.
- For any $m, n, p > 0$, describe a derivation of $b^m a b^n a b^p$.

4. **Constructing Grammars** Show that each of the following languages is context-free by giving a context-free grammar generating each:

- $\{w \in \{a, b, (,), +, *, \emptyset\}^* \mid w \text{ is a regular expression over } \{a, b\}^*\}$
(The definition of regular expressions given in class ignored the use of parentheses. For the purposes of this problem, if R is a regular expression then so is (R) .)
- $\{a^m b^n c^p d^q \mid m + n = p + q\}$
- $\{a^m b^n \mid m \leq 2n\}$