Lecture 11: Recursion

CS 51P

October 14, 2019

Recursion

EVE

CIVIC

MADAM

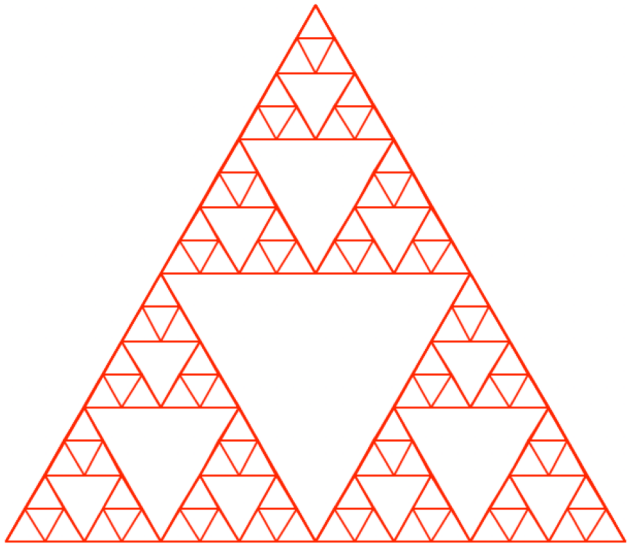
AVID DIVA

STEP ON NO PETS

STRESSED DESSERTS

ABLE WAS I ERE I SAW ELBA

LIVED ON DECAF FACED NO DEVIL



1, 1, 2, 3, 5, 8, 13, 21, ...

Writing Recursive Functions

0) Come up with a recursive definition of your problem
   - a _x_ is a _y_ plus _n_ smaller _x_s, unless it is really small, in which case it is _z_

1) Define a base case:
   - Some conditions when the function doesn't recursively call the function (the "really small" case)

2) Define a recursive case:
   - Some conditions when the function recursively calls itself
   - Make sure it calls itself with different (usually "smaller") arguments

3) Define the return value:
   - Combine answers from recursive call(s) into answer for full problem

Review: Fibonacci Numbers

Fibonacci (Leonardo Pisano)
1170-1240?

Fibonacci sequence: 0 1 1 2 3 5 8 13 …

$$F_n = \frac{\varphi^n - \psi^n}{\varphi - \psi} = \frac{\varphi^n - \psi^n}{\sqrt{5}}$$

Where $\varphi = \frac{1+\sqrt{5}}{2}$ and $\psi = 1 - \varphi$

Recursive definition:

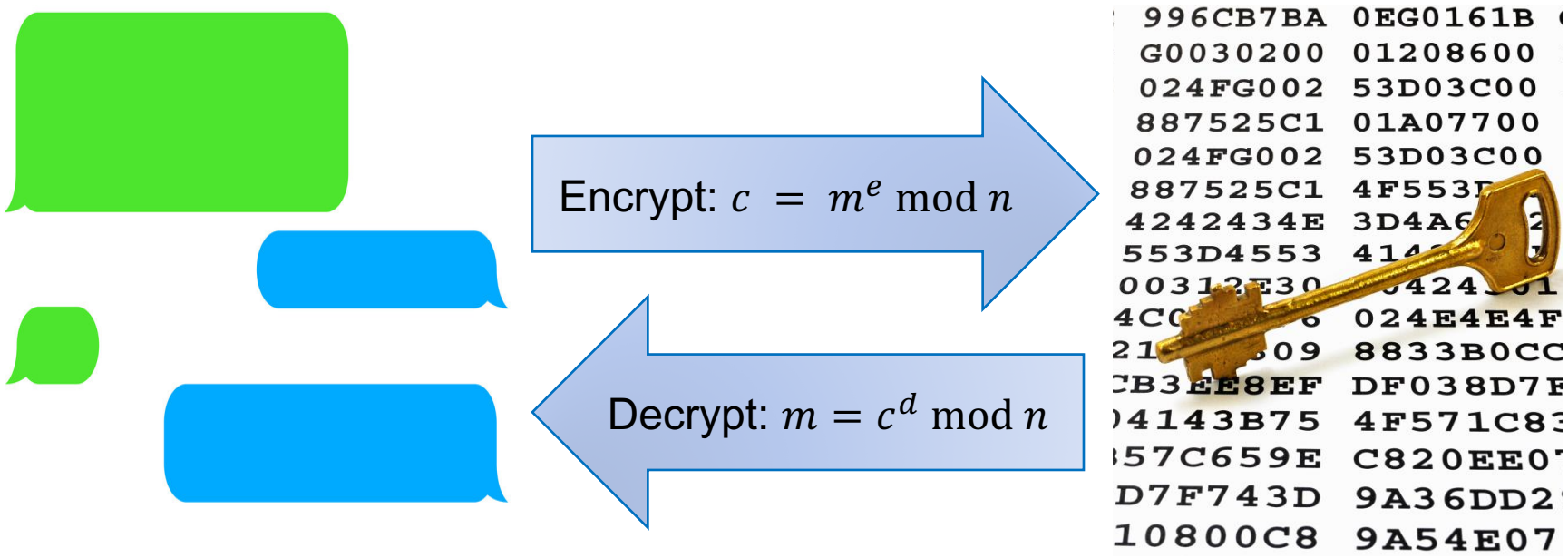
fib(0) = 0
fib(1) = 1 ← base cases

fib(n) = fib(n – 1) + fib(n – 2) n ≥ 2

Exponentiation

- $x^n = x \cdot x \cdot \ldots \cdot x$

Cryptography



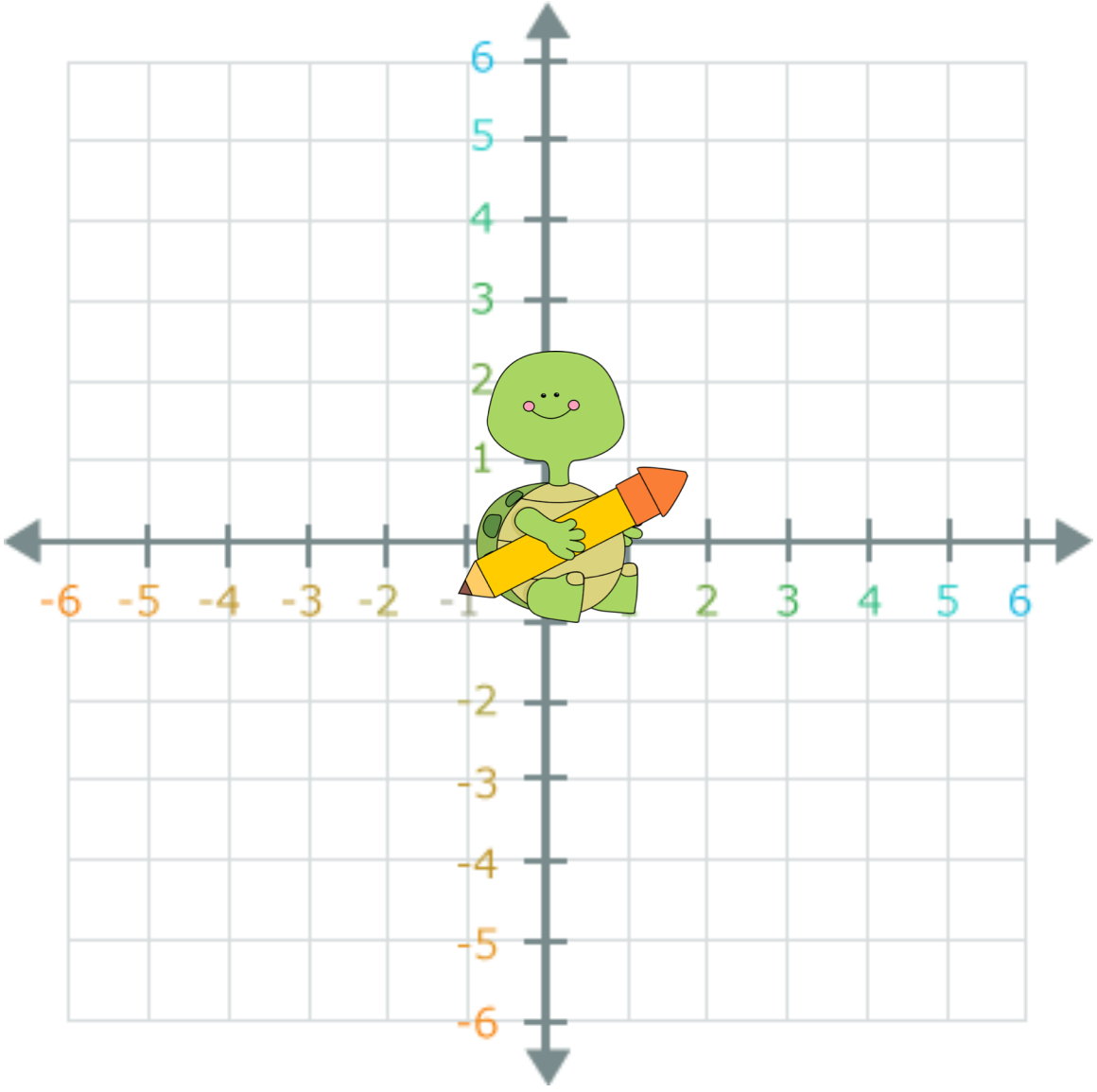
Exponentiation

- $x^n = x \cdot x \cdot \ldots \cdot x$

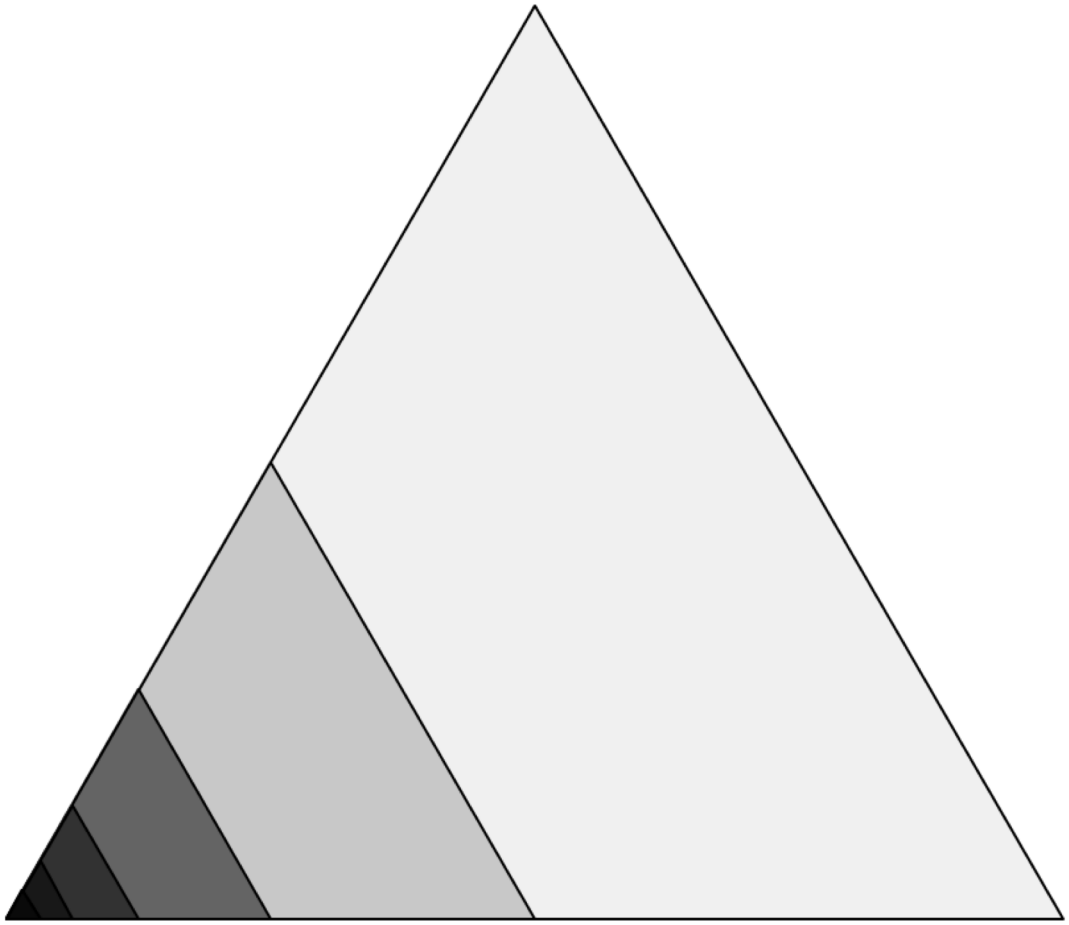
OR

- $x^n = (x^2)^{\frac{n}{2}}$ (if n is even)
- $x^n = x \cdot (x^2)^{\frac{(n-1)}{2}}$ (if n is odd)

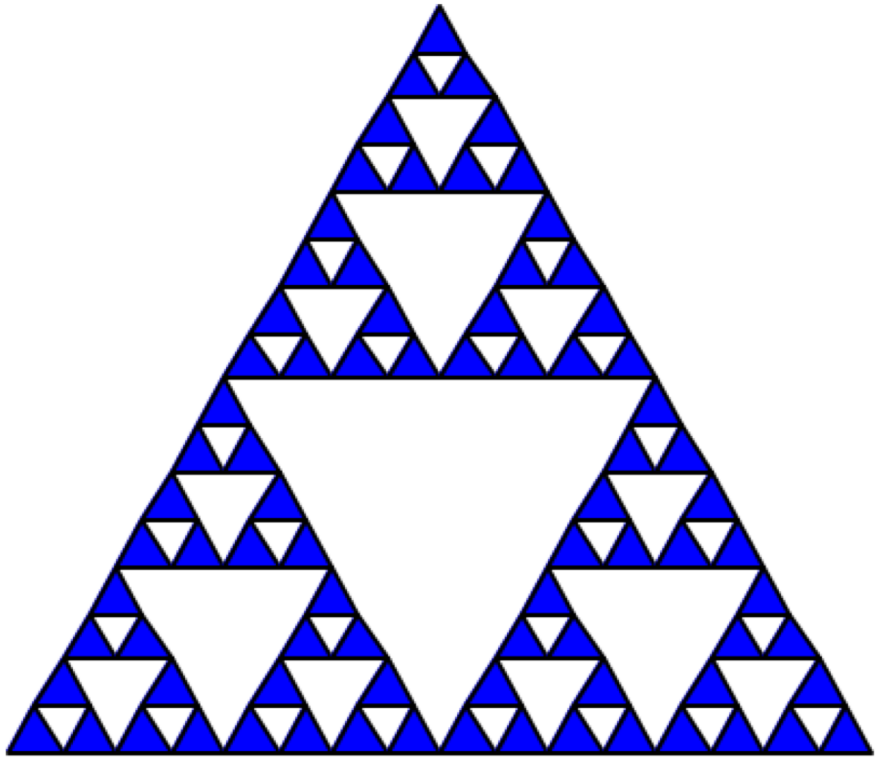
Turtle graphics



Recursive Graphics



Counting Triangles



Triangle Pyramid