

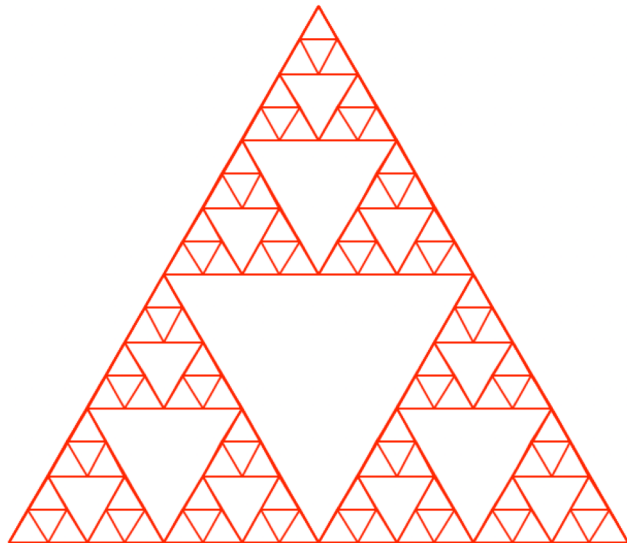
# Lecture 11: Recursion

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CS 51P

October 14, 2019

# Recursion



EVE  
CIVIC  
MADAM  
AVID DIVA  
STEP ON NO PETS  
STRESSED DESSERTS  
ABLE WAS I ERE I SAW ELBA  
LIVED ON DECAF FACED NO DEVIL

1, 1, 2, 3, 5, 8, 13, 21, ...

# Writing Recursive Functions

- 0) Come up with a recursive definition of your problem
  - a x is a y plus n smaller xs, unless it is really small, in which case it is z
- 1) Define a base case:
  - Some conditions when the function doesn't recursively call the function (the "really small" case)
- 2) Define a recursive case:
  - Some conditions when the function recursively calls itself
  - Make sure it calls itself with different (usually "smaller") arguments
- 3) Define the return value:
  - Combine answers from recursive call(s) into answer for full problem

# Review: Fibonacci Numbers

Fibonacci sequence: 0 1 1 2 3 5 8 13 ...

$$F_n = \frac{\varphi^n - \psi^n}{\varphi - \psi} = \frac{\varphi^n - \psi^n}{\sqrt{5}}$$

Where  $\varphi = \frac{1+\sqrt{5}}{2}$  and  $\psi = 1 - \varphi$

Recursive definition:

$$\text{fib}(0) = 0 \quad \leftarrow \text{base cases}$$

$$\text{fib}(1) = 1 \quad \leftarrow$$

$$\text{fib}(n) = \text{fib}(n - 1) + \text{fib}(n - 2) \quad n \geq 2$$

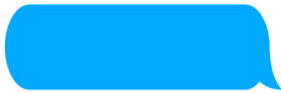


Fibonacci (Leonardo Pisano)  
1170-1240?

# Exponentiation

- $x^n = x \cdot x \cdot \dots \cdot x$

# Cryptography



Encrypt:  $c = m^e \text{ mod } n$

Decrypt:  $m = c^d \text{ mod } n$

996CB7BA	0EG0161B
G0030200	01208600
024FG002	53D03C00
887525C1	01A07700
024FG002	53D03C00
887525C1	4F553F
4242434E	3D4A6
553D4553	414
00312E30	5424
4CC	024E4E4F
21	8833B0CC
CB3EE8EF	DF038D7E
04143B75	4F571C83
57C659E	C820EE0
D7F743D	9A36DD2
10800C8	9A54E07



# Exponentiation

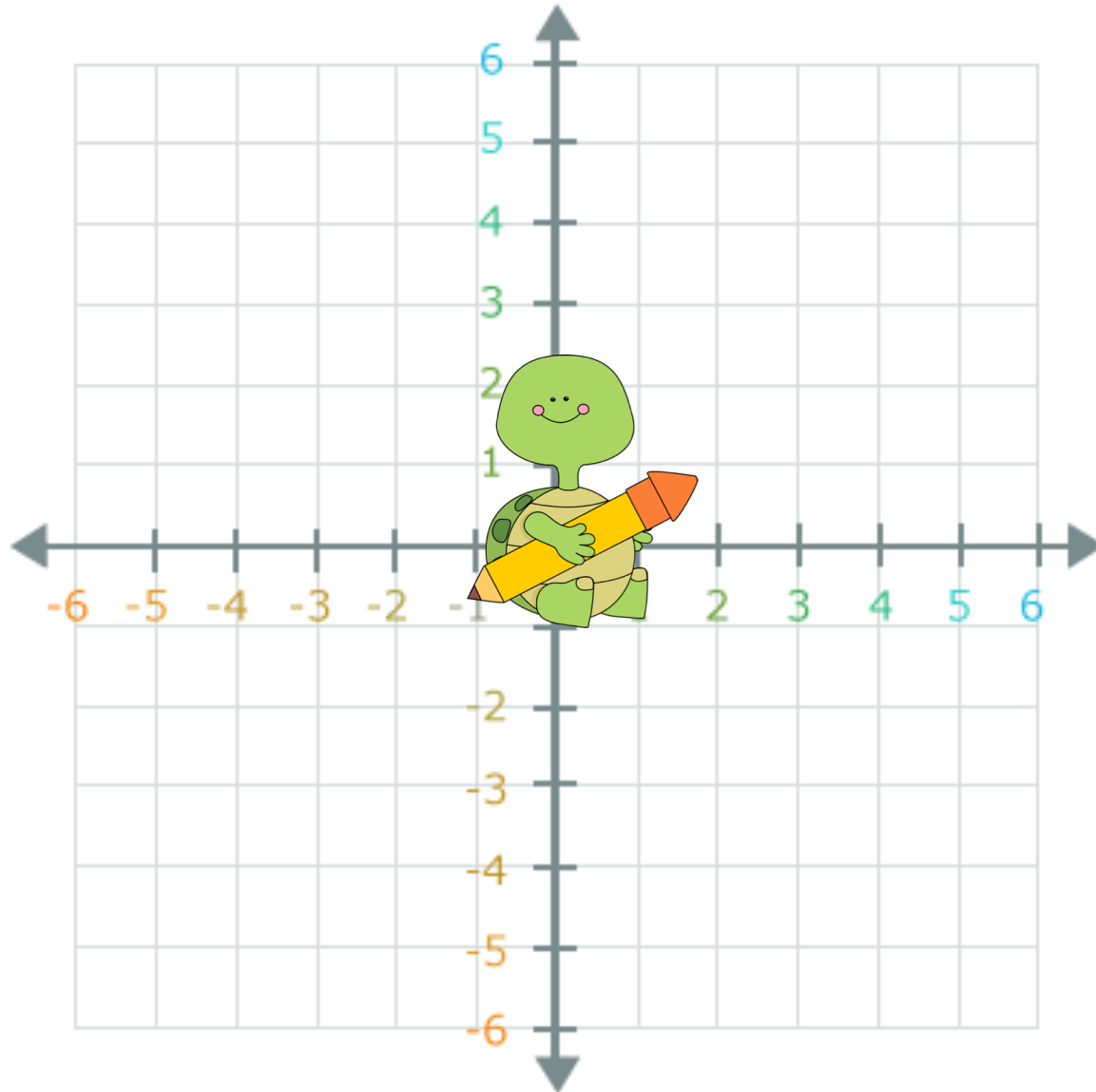
- $x^n = x \cdot x \cdot \dots \cdot x$

**OR**

- $x^n = (x^2)^{\frac{n}{2}}$  (if  $n$  is even)

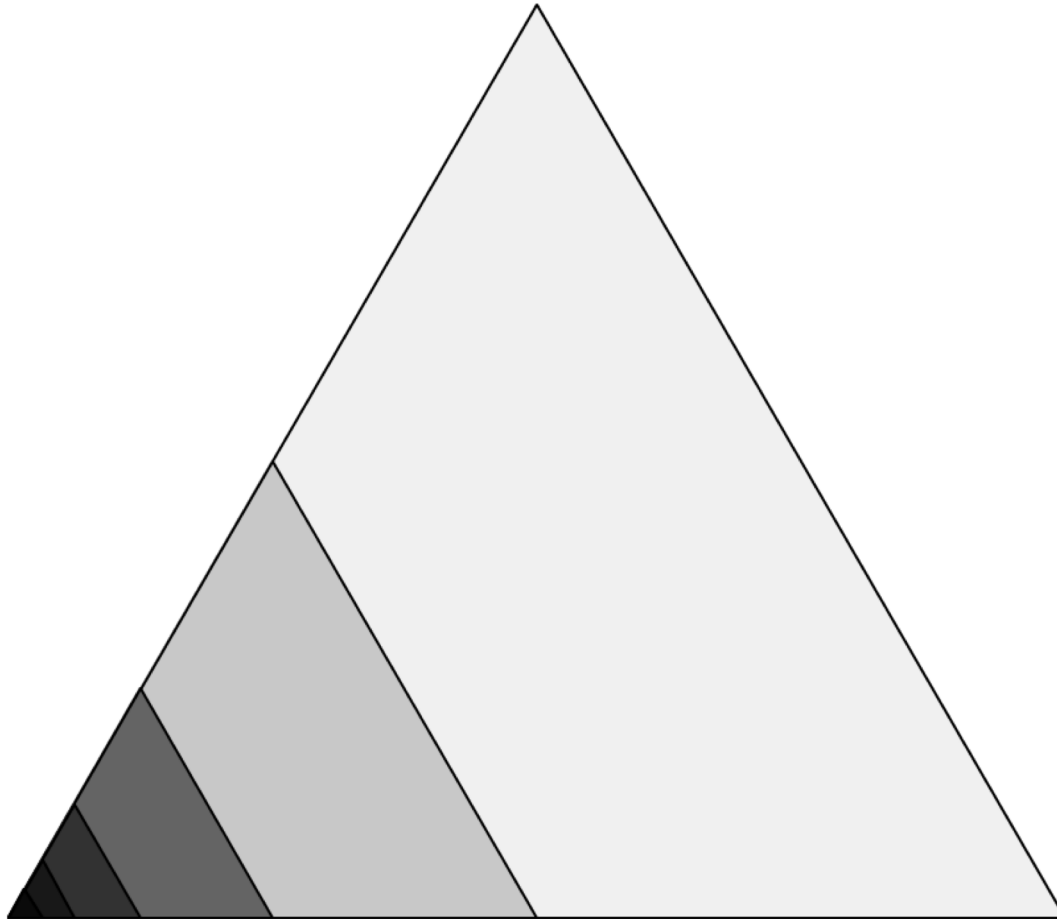
- $x^n = x \cdot (x^2)^{\frac{(n-1)}{2}}$  (if  $n$  is odd)

# Turtle graphics

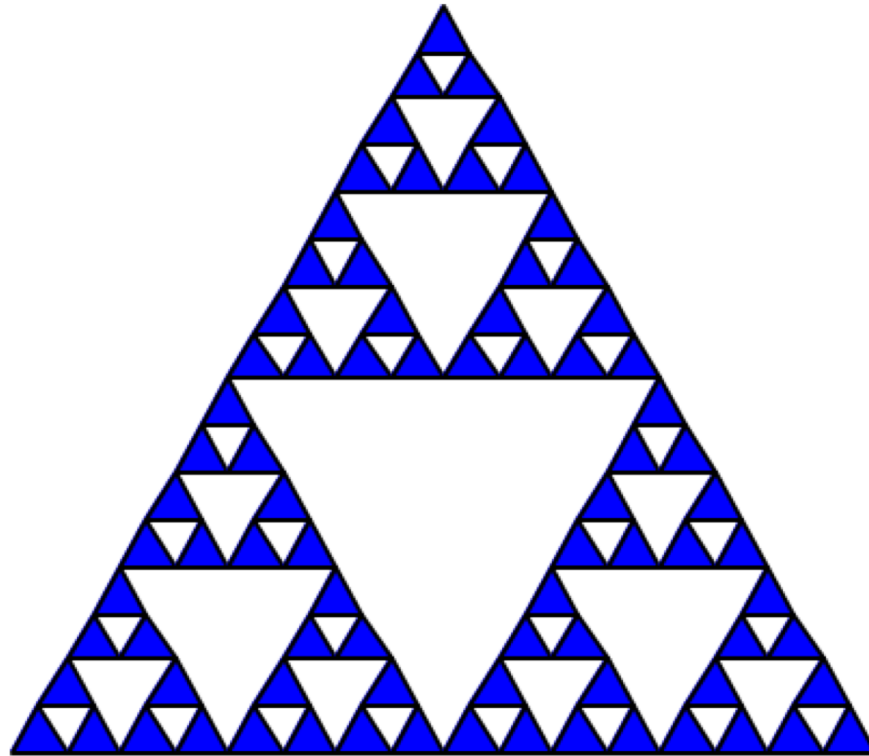




# Recursive Graphics



# Counting Triangles



# Triangle Pyramid

