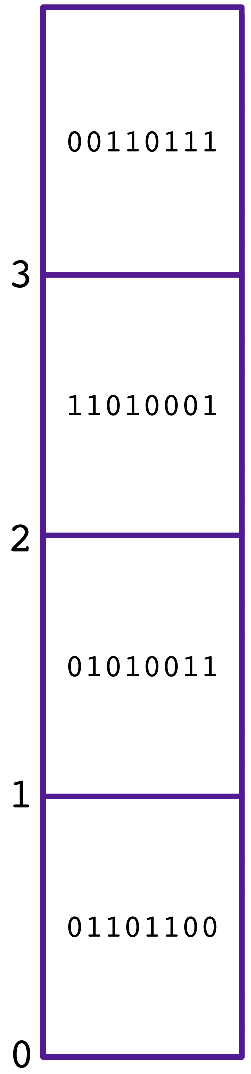
Lecture 2: Representing Integers

CS 105

Review: Abstraction

Review: Memory

- **Memory** is an array of ~~bits~~ bytes
- A **byte** is a unit of eight bits
- An index into the array is an **address**, **location**, or **pointer**
  - Often expressed in hexadecimal
- We speak of the *value* in memory at an address
  - The value may be a single byte …
  - … or a multi-byte quantity starting at that address



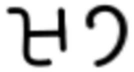
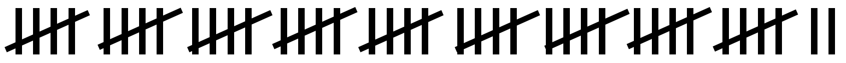
Review: Bits Require Interpretation

10001100 00001100 10101100 00000000

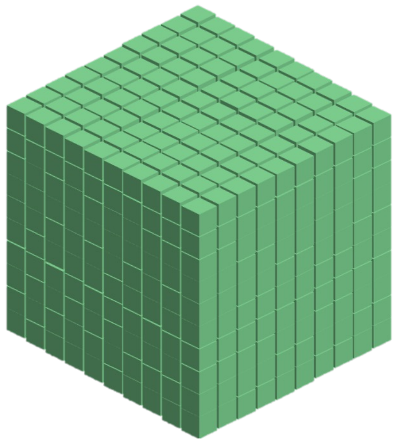
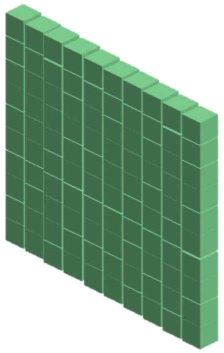
might be interpreted as

- The integer 3,485,745
- A floating point number close to 4.884569 x 10^{-39}
- The string “105”
- A portion of an image or video
- An address in memory

Representing Integers

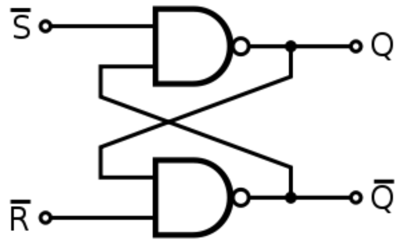
- Arabic Numerals: 47
- Roman Numerals: XLVII
- Brahmi Numerals: 
- Tally Marks: 

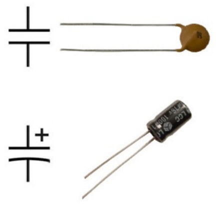
Base-10 Integers

1000 (10^3)	100 (10^2)	10 (10^1)	1 (10^0)
			
0	0	0	5
0	0	4	7
1	8	8	7

Storing bits

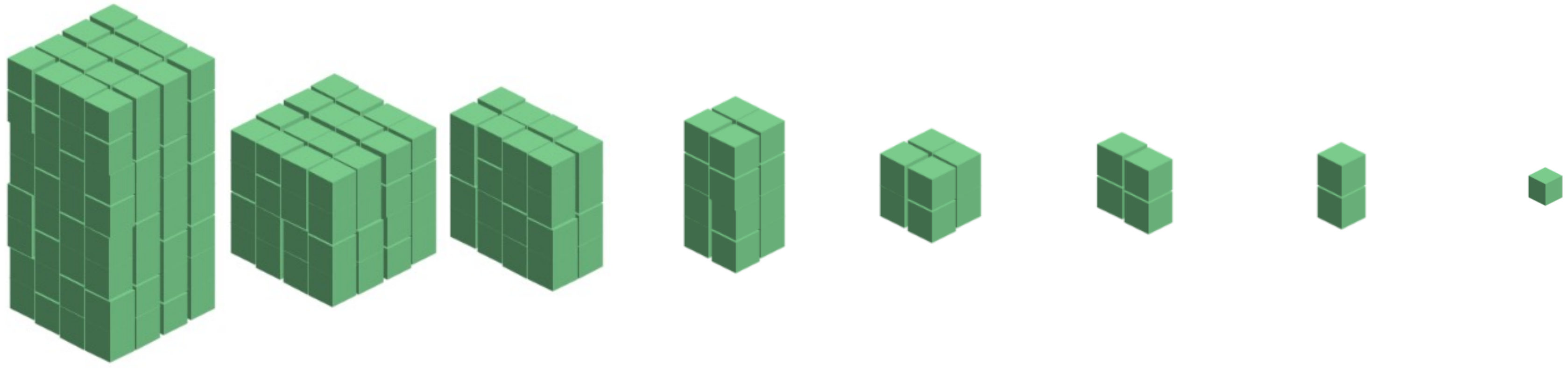
- Static random access memory (SRAM): stores each bit of data in a flip-flop, a circuit with two stable states
- Dynamic Memory (DRAM): stores each bit of data in a capacitor, which stores energy in an electric field (or not)
- Magnetic Disk: regions of the platter are magnetized with either N-S polarity or S-N polarity
- Optical Disk: stores bits as tiny indentations (pits) or not (lands) that reflect light differently
- Flash Disk: electrons are stored in one of two gates separated by oxide layers





Base-2 Integers (aka Binary Numbers)

128 (2^7)	64 (2^6)	32 (2^5)	16 (2^4)	8 (2^3)	4 (2^2)	2 (2^1)	1 (2^0)



0	0	0	0	0	1	0	1
0	0	1	0	1	1	1	1
1	1	1	1	1	1	1	1

Binary Numbers

- Decimal (Base-10):

1011

$$= 1 \cdot 10^3 + 0 \cdot 10^2 + 1 \cdot 10^1 + 1 \cdot 10^0$$
$$= 1011$$

- Binary (Base-2):

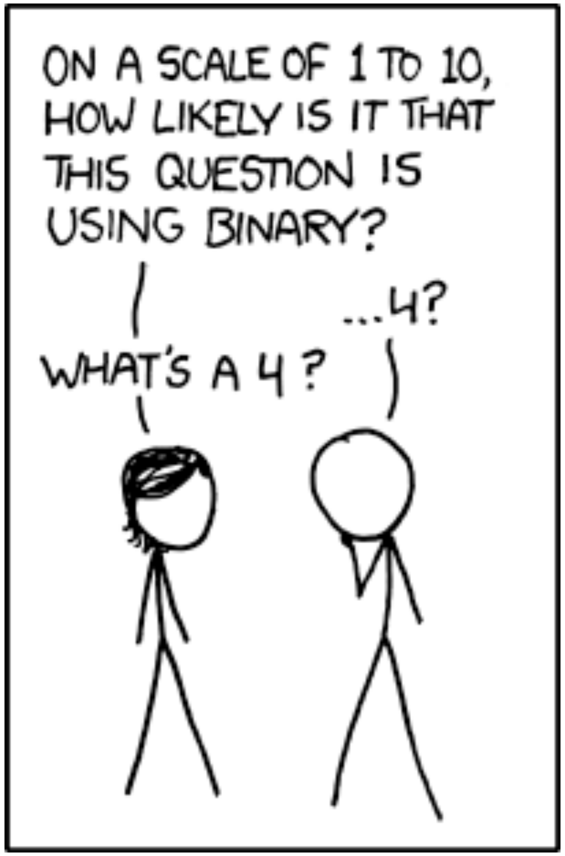
1011

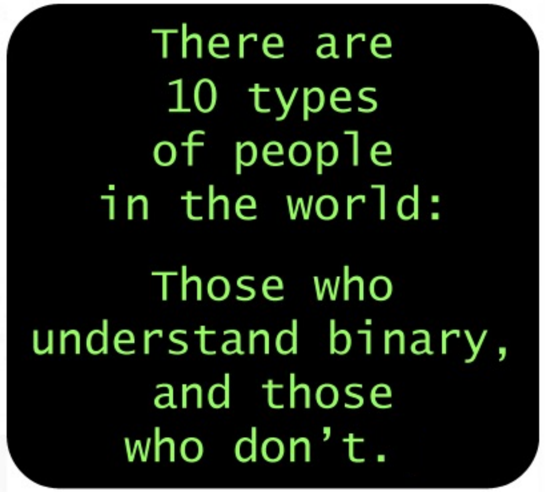
$$= 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$$
$$= 11$$

Exercise 1: Binary Numbers

- Consider the following four-bit binary values. What is the (base-10) integer interpretation of these values?
  1. 0001
  2. 1010
  3. 0111
  4. 1111

Binary Numbers


ON A SCALE OF 1 TO 10, HOW LIKELY IS IT THAT THIS QUESTION IS USING BINARY?
WHAT'S A 4?
...4?


There are
10 types
of people
in the world:

Those who
understand binary,
and those
who don't.

Exercise 2: Binary Number Range

- What are the max number and min number that can be represented by a w-bit binary number?

1. w = 3
2. w = 4
3. w = 8

Unsigned Integers in C

C Data Type	Size (bytes)
unsigned char	1
unsigned short	2
unsigned int	4
unsigned long	8

ASCII characters

Char	Dec	Binary	Char	Dec	Binary	Char	Dec	Binary	Char	Dec	Binary	Char	Dec	Bi
!	33	00100001	1	49	00110001	A	65	01000001	Q	81	01010001	a	97	0110
"	34	00100010	2	50	00110010	B	66	01000010	R	82	01010010	b	98	0110
#	35	00100011	3	51	00110011	C	67	01000011	S	83	01010011	c	99	0110
$	36	00100100	4	52	00110100	D	68	01000100	T	84	01010100	d	100	0110
%	37	00100101	5	53	00110101	E	69	01000101	U	85	01010101	e	101	0110
&	38	00100110	6	54	00110110	F	70	01000110	V	86	01010110	f	102	0110
'	39	00100111	7	55	00110111	G	71	01000111	W	87	01010111	g	103	0110
(	40	00101000	8	56	00111000	H	72	01001000	X	88	01011000	h	104	0110
)	41	00101001	9	57	00111001	I	73	01001001	Y	89	01011001	i	105	0110
*	42	00101010	:	58	00111010	J	74	01001010	Z	90	01011010	j	106	0110
+	43	00101011	;	59	00111011	K	75	01001011	[	91	01011011	k	107	0110
,	44	00101100	<	60	00111100	L	76	01001100	\	92	01011100	l	108	0110
-	45	00101101	=	61	00111101	M	77	01001101	]	93	01011101	m	109	0110
.	46	00101110	>	62	00111110	N	78	01001110	^	94	01011110	n	110	0110
/	47	00101111	?	63	00111111	O	79	01001111	_	95	01011111	o	111	0110
0	48	00110000	@	64	01000000	P	80	01010000	`	96	01100000	p	112	0111

Hexidecimal Numbers

$$\boxed{\underbrace{\texttt{0010}}_{\texttt{2}}\underbrace{\texttt{1100}}_{\texttt{c}}}\boxed{\underbrace{\texttt{0011}}_{\texttt{3}}\underbrace{\texttt{0101}}_{\texttt{5}}}\boxed{\underbrace{\texttt{0011}}_{\texttt{3}}\underbrace{\texttt{0000}}_{\texttt{0}}}\boxed{\underbrace{\texttt{1110}}_{\texttt{e}}\underbrace{\texttt{0001}}_{\texttt{1}}}$$

`0x2c3530e1`

Dec	Hex
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	a
11	b
12	c
13	d
14	e
15	f

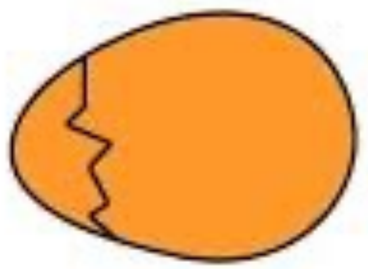
Exercise 3: Hexidecimal Numbers

- Consider the following hexidecimal values. What is the representation of each value in (1) binary and (2) decimal?
  1. 0x0a
  2. 0x11
  3. 0x2f

Endianness

47 vs 74

BIG ENDIAN - The way people always broke their eggs in the Lilliput land



LITTLE ENDIAN - The way the king then ordered the people to break their eggs

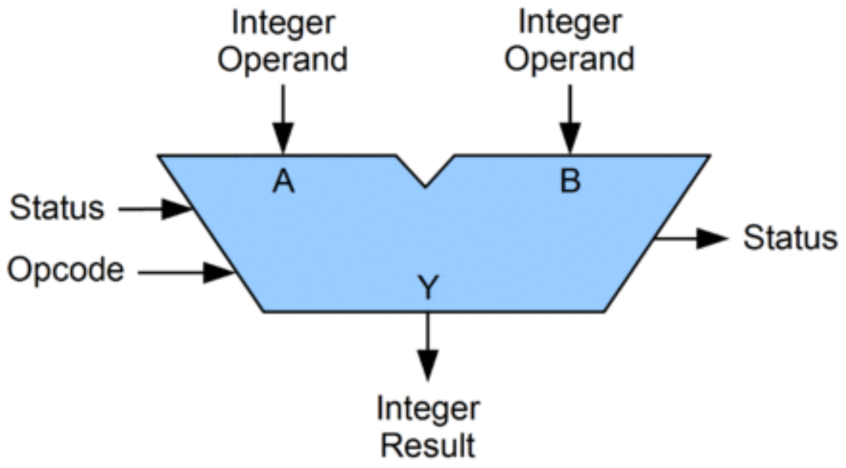
Endianness

- **Big Endian:** low-order bits go on the right (47)
  - I tend to think in big endian numbers, so examples in class will generally use this representation
  - Networks generally use big endian (aka network byte order)
- **Little Endian:** low-order bits go on the left (74)
  - Most modern machines use this representation

- I will try to always be clear about whether I'm using a big endian or little endian representation
- When in doubt, ask!

Arithmetic Logic Unit (ALU)

- circuit that performs bitwise operations and arithmetic on integer binary types



Bitwise vs Logical Operations in C

- Bitwise Operators &, |, ~, ^
  - View arguments as bit vectors
  - operations applied bit-wise in parallel
- Logical Operators `&&`, `||`, `!`
  - View 0 as "False"
  - View anything nonzero as "True"
  - Always return 0 or 1
  - Early termination
- Shift operators `<<`, `>>`
  - Left shift fills with zeros
  - For unsigned integers, right shift is logical (fills with zeros)

Exercise 4: Bitwise vs Logical Operations

Assume unsigned char data type (one byte). What do each of the following expressions evaluate to (interpreted as unsigned integers and expressed base-10)?

1. `~226`
2. `!226`

3. `120 &  85`
4. `120 |  85`
5. `120 && 85`
6. `120 || 85`

7. `81 << 2`
8. `81 >> 2`

Example: Using Bitwise Operations

• `x & 1`	“x is odd”
• `(x + 7) & 0xFFFFFFF8`	“round up to a multiple of 8”
• `x << 2`	"multiply by 4"

Addition Example

- Compute 5 + 6 assuming all ints are stored as eight-bit (1 byte) unsigned values

$$\begin{array}{ccccccccc}
 & & & & & 1 & & & \\
 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
+ & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
\hline
 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1
\end{array} \quad = 11 \text{ (Base-10)}$$

Like you learned in grade school, only binary!

… and with a finite number of digits

Addition Example with Overflow

- Compute 200 + 100 assuming all ints are stored as eight-bit (1 byte) unsigned values

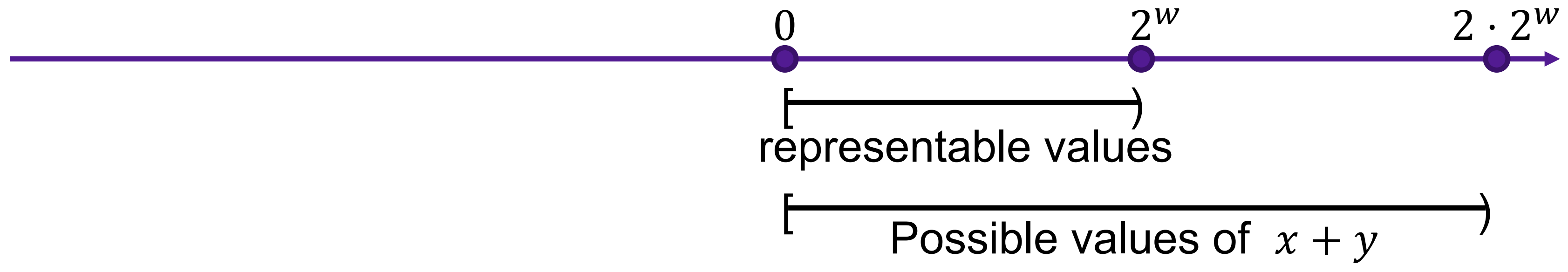
$$\begin{array}{ccccccccccl}
1 & 1 & & & & & & & & & \\
 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & & \\
+ & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & & \\
\hline
 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & & \text{= 44 (Base-10)}
\end{array}$$

Like you learned in grade school, only binary!

… and with a finite number of digits

Error Cases

- Assume w-bit unsigned values



- $x +_w^u y = \begin{cases} x + y & \text{(normal)} \\ x + y - 2^w & \text{(overflow)} \end{cases}$

- overflow has occurred iff $x +_w^u y < x$

Exercise 5: Binary Addition

- Given the following 5-bit unsigned values, compute their sum and indicate whether or not an overflow occurred

x	y	x+y	overflow?
00010	00101		
01100	00100		
10100	10001		

Multiplication Example

- Compute 5 x 6 assuming all ints are stored as eight-bit (1 byte) unsigned values

```
      0 0 0 0 0 1 0 1
    x 0 0 0 0 0 1 1 0
    -----------------
      0 0 0 0 0 0 0 0
    0 0 0 0 0 1 0 1 0
 +0 0 0 0 0 1 0 1 0 0
 --------------------
      0 0 0 1 1 1 1 0   = 30 (Base-10)
```

Like you learned in grade school, only binary!

… and with a finite number of digits

Multiplication Example

- Compute 200 x 3 assuming all ints are stored as eight-bit (1 byte) unsigned values

$$
\begin{array}{r}
1\,1\,0\,0\,1\,0\,0\,0 \\
\times\ 0\,0\,0\,0\,0\,0\,1\,1 \\ \hline
1\,1\,0\,0\,1\,0\,0\,0 \\
+\ \ 1\,1\,0\,0\,1\,0\,0\,0\,0 \\ \hline
1\,0\,0\,1\,0\,1\,1\,0\,0\,0
\end{array}
$$

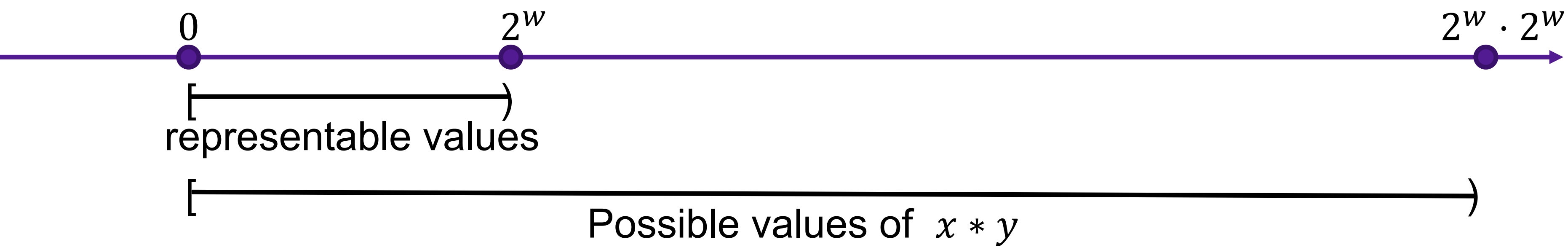
= 88 (Base-10)

Like you learned in grade school, only binary!

… and with a finite number of digits

Error Cases

- Assume w-bit unsigned values



- $x *_w^u y = (x \cdot y) \bmod 2^w$

Exercise 6: Binary Multiplication

- Given the following 3-bit unsigned values, compute their product and indicate whether or not an overflow occurred

x	y	x*y	overflow?
100	101		
010	011		
111	010		

Multiplying with Shifts

- Multiplication is slow
- Bit shifting is kind of like multiplication, and is often faster
  - x * 8 = x << 3
  - x * 10 = x << 3 + x << 1
- Most compilers will automatically replace multiplications with shifts where possible