CS105 – Computer Systems Fall 2020

Problem Session 1: Binary Numbers and Operations

SOLUTION

Wednesday, August 26, 2020

1. Consider a **5-bit** unsigned integer representation. Fill in the empty boxes in the following table. Addition and subtraction should be performed based on the rules for 5-bit, unsigned integer arithmetic.

Expression	Decimal Representation	Binary Representation
`13`	13	`01101`
`21`	21	`10101`
n/a	10	`01010`
n/a	19	`10011`
`13 & 21`	5	`00101`
`13 && 21`	1	`00001`
`13 | 21`	29	`11101`
`13 || 21`	1	`00001`
`13 ^ 21`	24	`11000`
`~13`	18	`10010`
`!21`	0	`00000`
`13 << 3`	8	`01000`
`21 >> 1`	10	`01010`
`13 + 21`	2	`00010`
`13 * 21`	17	`10001`

2. In the following questions assume the variables `a` and `b` are unsigned 32-bit integers. Also assume that `UMAX` is the maximum unsigned 32-bit integer, `UMIN` is the minimum integer, and `W` is one less than the word length (i.e.., `W` = 31, since we're dealing with 32-bit integers).

   Match each of the descriptions on the left with a line of code on the right (write in the letter).

1. `a`

   b

2. `~a`

   e

3. `a & b`

   a

4. `a * 5`

   d

5. `a / 4`

   f

a. `~(~a | (b ^ UMAX))`

b. `((a ^ b) & ~b) | (~(a ^ b) & b)`

c. `1 + (a << 3) + ~a`

d. `(a << 2) + a`

e. `a ^ (UMIN + UMAX)`

f. `a >> 2`

g. `(a << 4) + (a << 1)`

3. For each of the following expressions, write an equivalent expression using only the allowed operations:

   (a) Write an expression that evaluates to `x | y` using only the operations `&, ~`

   ```
   ~( ~x & ~y)
   ```

   (b) Write an expression that evaluates to `x ^ y` using only the operations `&, ~`

   ```
   (~(x & y)) & (~(~x & ~y))
   ```

   (c) Write an expression that evaluates to `x == y` using the operations `&, |, ^, ~, &&, ||, !` Recall that `x == y` evaluates to `1` if the values are equal and `0` otherwise.

   ```
   !(x ^ y)
   ```