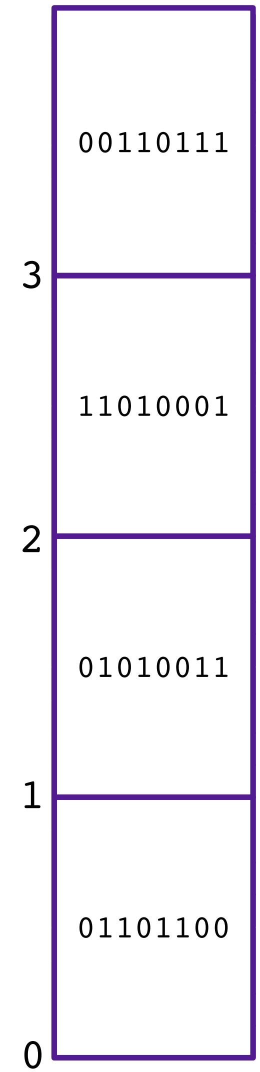
Lecture 2: Representing Integers

CS 105 Fall 2020

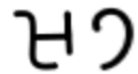
Abstraction

Memory: A (very large) array of bytes

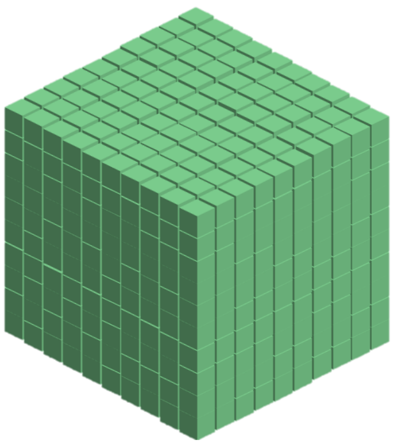
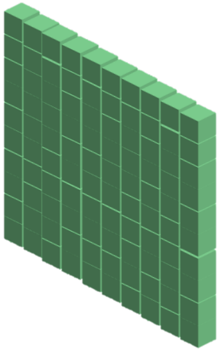
- **Memory** is an array of ~~bits~~ bytes
- A **byte** is a unit of eight bits
- An index into the array is an **address**, **location**, or **pointer**
  - Often expressed in hexadecimal
- We speak of the *value* in memory at an address
  - The value may be a single byte …
  - … or a multi-byte quantity starting at that address



Representing Integers

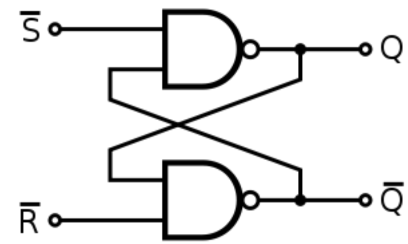
- Arabic Numerals: 47
- Roman Numerals: XLVII
- Brahmi Numerals: 
- Tally Marks: 

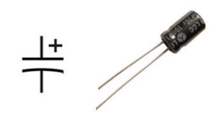
Base-10 Integers

1000 (10^3)	100 (10^2)	10 (10^1)	1 (10^0)
			
0	0	0	5
0	0	4	7
1	8	8	7

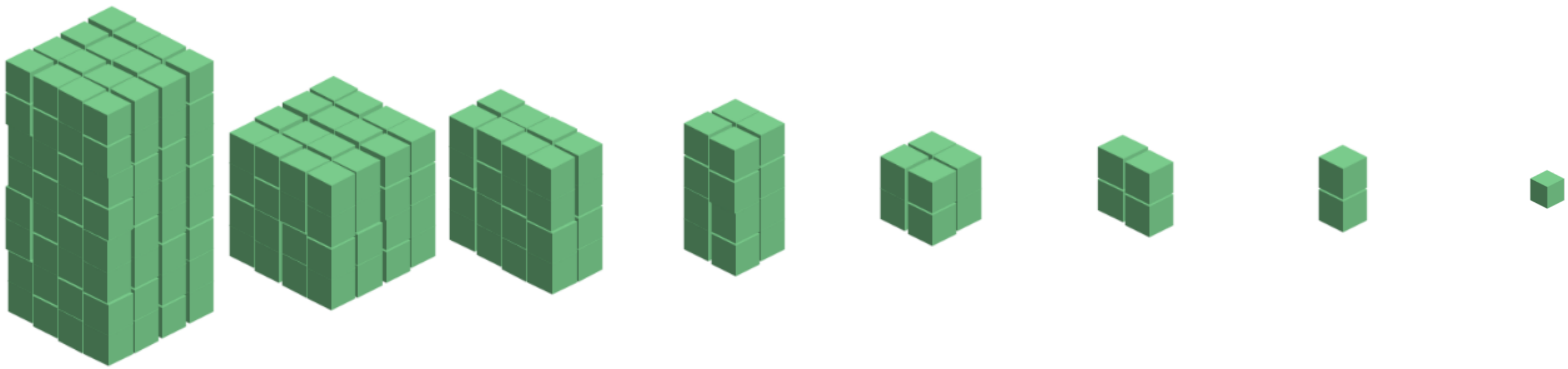
Storing bits

- Static random access memory (SRAM): stores each bit of data in a flip-flop, a circuit with two stable states
- Dynamic Memory (DRAM): stores each bit of data in a capacitor, which stores energy in an electric field (or not)
- Magnetic Disk: regions of the platter are magnetized with either N-S polarity or S-N polarity
- Optical Disk: stores bits as tiny indentations (pits) or not (lands) that reflect light differently
- Flash Disk: electrons are stored in one of two gates separated by oxide layers





Base-2 Integers (aka Binary Numbers)



128 (2^7)	64 (2^6)	32 (2^5)	16 (2^4)	8 (2^3)	4 (2^2)	2 (2^1)	1 (2^0)
0	0	0	0	0	1	0	1
0	0	1	0	1	1	1	1
1	1	1	1	1	1	1	1

Binary Numbers

- Decimal (Base-10):

$$4211$$

$$= 4 \cdot 10^3 + 2 \cdot 10^2 + 1 \cdot 10^1 + 1 \cdot 10^0$$
$$= 4211$$

- Binary (Base-2):

$$1011$$

$$= 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0$$
$$= 11$$

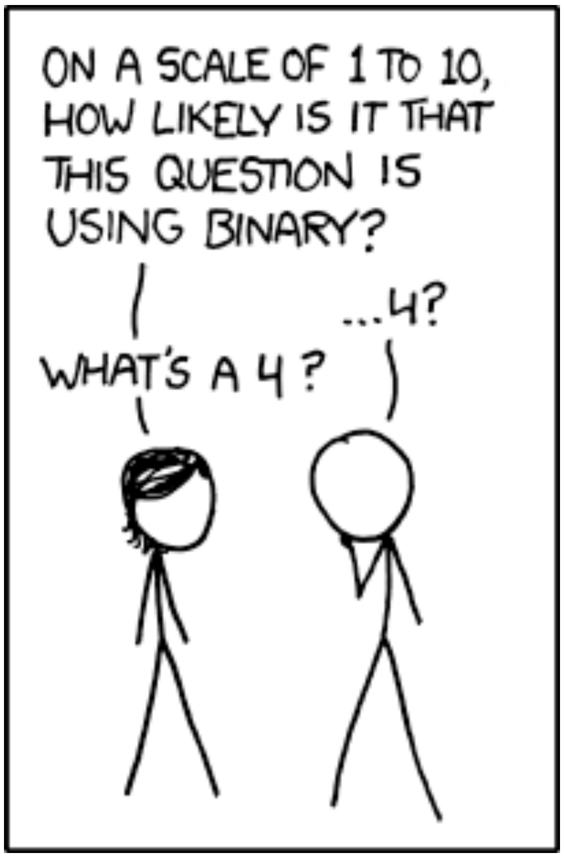
Exercise 1: Binary Numbers

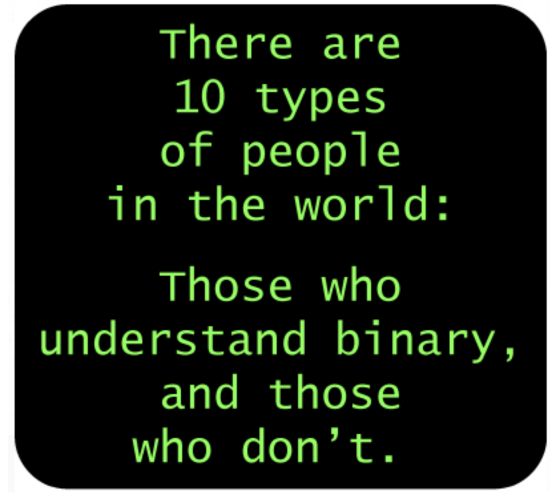
- Consider the following four-bit binary values. What is the (base-10) integer interpretation of these values?
  1. 0001
  2. 1010
  3. 0111
  4. 1111

Exercise 1: Binary Numbers

- Consider the following four-bit binary values. What is the (base-10) integer interpretation of these values?
  1. 0001 $= 0 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^1 = 1$
  2. 1010 $= 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^1 = 8 + 2 = 10$
  3. 0111 $= 0 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^1 = 4 + 2 + 1 = 7$
  4. 1111 $= 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^1 = 8 + 4 + 2 + 1 = 15$

Binary Numbers





Exercise 2: Binary Number Range

- What are the max number and min number that can be represented by a w-bit binary number?
  1. w = 3
  2. w = 4
  3. w = 8

Exercise 2: Binary Number Range

- What are the max number and min number that can be represented by a w-bit binary number?

1. w = 3 $\quad \text{min} = 000_2 = 0_{10} \quad \text{max} = 111_2 = 2^2 + 2^1 + 2^0 = 7_{10}$
2. w = 4 $\quad \text{min} = 0000_2 = 0_{10} \quad \text{max} = 1111_2 = 2^3 + 2^2 + 2^1 + 2^0 = 15_{10}$
3. w = 8 $\quad \text{min} = 00000000_2 = 0_{10} \quad \text{max} = 11111111_2 = 2^7 + 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0 = 255_{10}$

Unsigned Integers in C

C Data Type	Size (bytes)
unsigned char	1
unsigned short	2
unsigned int	4
unsigned long	8

ASCII characters

Char	Dec	Binary	Char	Dec	Binary	Char	Dec	Binary	Char	Dec	Binary	Char	Dec	Bi
!	33	00100001	1	49	00110001	A	65	01000001	Q	81	01010001	a	97	0110
"	34	00100010	2	50	00110010	B	66	01000010	R	82	01010010	b	98	0110
#	35	00100011	3	51	00110011	C	67	01000011	S	83	01010011	c	99	0110
$	36	00100100	4	52	00110100	D	68	01000100	T	84	01010100	d	100	0110
%	37	00100101	5	53	00110101	E	69	01000101	U	85	01010101	e	101	0110
&	38	00100110	6	54	00110110	F	70	01000110	V	86	01010110	f	102	0110
'	39	00100111	7	55	00110111	G	71	01000111	W	87	01010111	g	103	0110
(	40	00101000	8	56	00111000	H	72	01001000	X	88	01011000	h	104	0110
)	41	00101001	9	57	00111001	I	73	01001001	Y	89	01011001	i	105	0110
*	42	00101010	:	58	00111010	J	74	01001010	Z	90	01011010	j	106	0110
+	43	00101011	;	59	00111011	K	75	01001011	[	91	01011011	k	107	0110
,	44	00101100	<	60	00111100	L	76	01001100	\	92	01011100	l	108	0110
-	45	00101101	=	61	00111101	M	77	01001101	]	93	01011101	m	109	0110
.	46	00101110	>	62	00111110	N	78	01001110	^	94	01011110	n	110	0110
/	47	00101111	?	63	00111111	O	79	01001111	_	95	01011111	o	111	0110
0	48	00110000	@	64	01000000	P	80	01010000	`	96	01100000	p	112	0111

Hexidecimal Numbers

$$\boxed{\underbrace{0010}_{2}\underbrace{1100}_{c}}\boxed{\underbrace{0011}_{3}\underbrace{0101}_{5}}\boxed{\underbrace{0011}_{3}\underbrace{0000}_{0}}\boxed{\underbrace{1110}_{e}\underbrace{0001}_{1}}$$

`0x2c3530e1`

Dec	Hex
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	a
11	b
12	c
13	d
14	e
15	f

Exercise 3: Hexidecimal Numbers

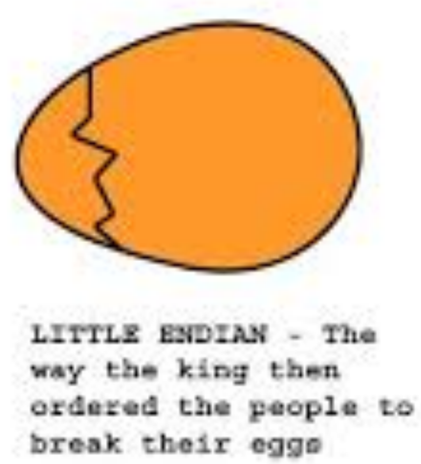
- Consider the following hexidecimal values. What is the representation of each value in (1) binary and (2) decimal?
  1. 0x0a
  2. 0x11
  3. 0x2f

Exercise 3: Hexidecimal Numbers

- Consider the following hexidecimal values. What is the representation of each value in (1) binary and (2) decimal?
  1. 0x0a $= 00001010_2 \quad = 10_{10}$
  2. 0x11 $= 00010001_2 \quad = 17_{10}$
  3. 0x2f $= 00101111_2 \quad = 47_{10}$

Endianness

47 vs 74



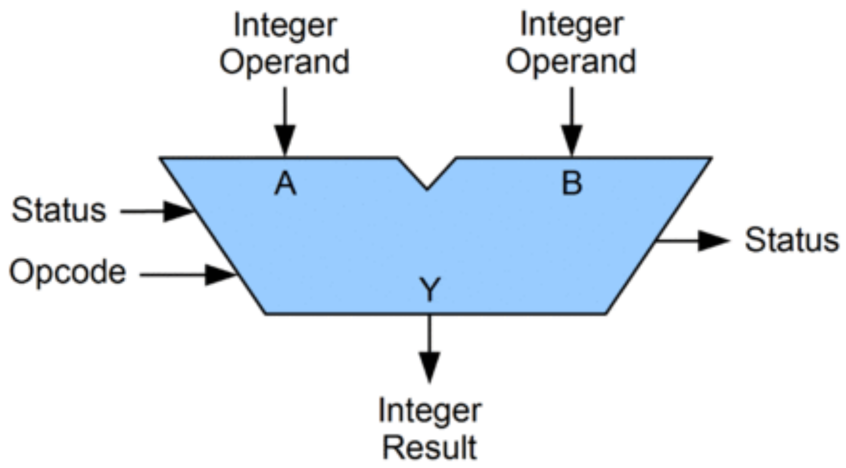
Endianness

- **Big Endian:** low-order bits go on the right (47)
  - I tend to think in big endian numbers, so examples in class will generally use this representation
  - Networks generally use big endian (aka network byte order)
- **Little Endian:** low-order bits go on the left (74)
  - Most modern machines use this representation

- I will try to always be clear about whether I'm using a big endian or little endian representation
- When in doubt, ask!

Arithmetic Logic Unit (ALU)

- circuit that performs bitwise operations and arithmetic on integer binary types



Bitwise vs Logical Operations in C

- Bitwise Operators &, |, ~, ^
  - View arguments as bit vectors
  - operations applied bit-wise in parallel

- Logical Operators &&, ||, !
  - View 0 as “False”
  - View anything nonzero as “True”
  - Always return 0 or 1
  - Early termination

- Shift operators <<, >>
  - Left shift fills with zeros
  - For unsigned integers, right shift is logical (fills with zeros)

Exercise 4: Bitwise vs Logical Operations

- Assume one byte (8 bit) unsigned integer values

  - `~226`
  - `!226`

  - `120 &  85`
  - `120 |  85`
  - `120 && 85`
  - `120 || 85`

- `81 << 4`
- `81 << 2`
- `81 >> 4`
- `81 >> 2`

Exercise 4: Bitwise vs Logical Operations

- Assume unsigned char data type (one byte)

  - `~226` = `~11100010 = 00011101 = 0x1d`
  - `!226` = `!11100010 = 00000000 = 0x00`

  - `120 &  85` = `01111000 &  01010101 = 01010000 = 0x50`
  - `120 |  85` = `01111000 |  01010101 = 01111101 = 0x7d`
  - `120 && 85` = `01111000 && 01010101 = 00000001 = 0x01`
  - `120 || 85` = `01111000 || 01010101 = 00000001 = 0x01`

  - `81 << 4` = `01010001 << 4 = 00010000 = 0x10`
  - `81 << 2` = `01010001 << 2 = 01000100 = 0x44`
  - `81 >> 4` = `01010001 >> 4 = 00000101 = 0x05`
  - `81 >> 2` = `01010001 >> 2 = 00010100 = 0x14`

Example: Using Bitwise Operations

- `x & 1` “x is odd”
- `(x + 7) & 0xFFFFFFF8` “round up to a multiple of 8”
- `x << 2` "multiply by 4"

Addition Example

- Compute 5 + 6 assuming all ints are stored as eight-bit (1 byte) unsigned values

$$
\begin{array}{r}
1 \\
00000101 \\
+\ 00000110 \\
\hline
00001011
\end{array}
\quad = 11 \text{ (Base-10)}
$$

Like you learned in grade school, only binary!

… and with a finite number of digits

Addition Example with Overflow

- Compute 200 + 100 assuming all ints are stored as eight-bit (1 byte) unsigned values

$$\begin{array}{cccccccccc}
 & 1 & 1 & & & & & & & \\
 & & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
+ & & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\
\hline
 & & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0
\end{array}$$

= 44 (Base-10)

Like you learned in grade school, only binary!

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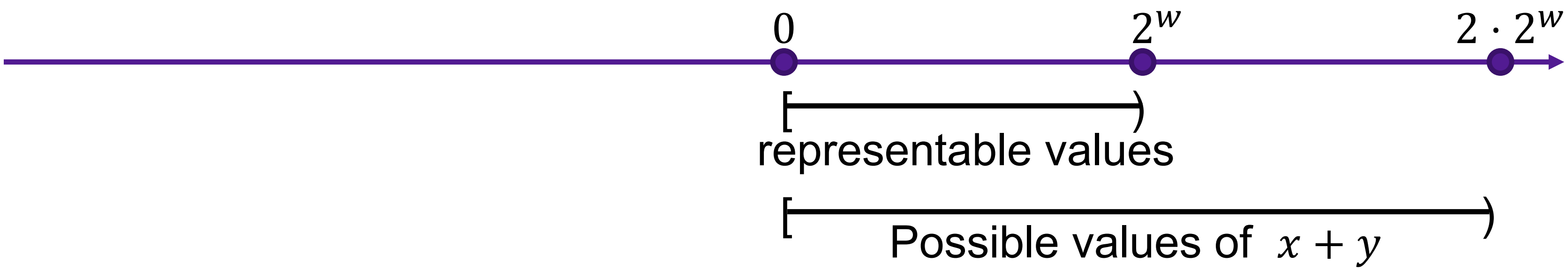
Error Cases

- Assume w-bit unsigned values



- $x +_w^u y = \begin{cases} x + y & \text{(normal)} \\ x + y - 2^w & \text{(overflow)} \end{cases}$

- overflow has occurred iff $x +_w^u y < x$

Exercise 5: Binary Addition

- Given the following 5-bit unsigned values, compute their sum and indicate whether or not an overflow occurred

x	y	x+y	overflow?
00010	00101		
01100	00100		
10100	10001		

Exercise 5: Binary Addition

- Given the following 5-bit unsigned values, compute their sum and indicate whether or not an overflow occurred

x	y	x+y	overflow?
00010	00101	**00111**	**no**
01100	00100	**10000**	**no**
10100	10001	**00101**	**yes**

Multiplication Example

- Compute 5 x 6 assuming all ints are stored as eight-bit (1 byte) unsigned values

$$
\begin{array}{r}
00000101 \\
\times\ 00000110 \\
\hline
00000000 \\
000001010 \\
+\ 0000010100 \\
\hline
00011110
\end{array}
\quad = 30 \text{ (Base-10)}
$$

Like you learned in grade school, only binary!

… and with a finite number of digits

Addition Example

- Compute 200 x 3 assuming all ints are stored as eight-bit (1 byte) unsigned values

$$
\begin{array}{rcccccccccc}
 & & & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
 & & \times & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
\hline
 & & & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
+ & & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\hline
 & 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0
\end{array}
\quad = 88 \text{ (Base-10)}
$$

Like you learned in grade school, only binary!

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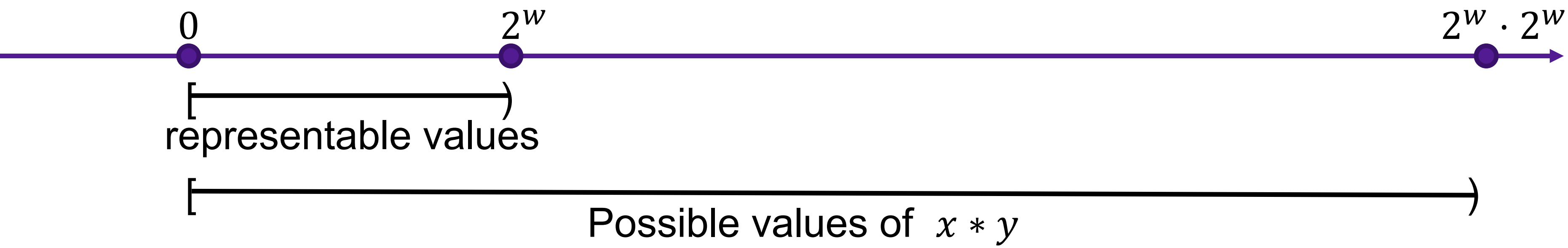
Error Cases

- Assume w-bit unsigned values



- $x *_w^u y = (x \cdot y) \bmod 2^w$

Exercise 6: Binary Multiplication

- Given the following 3-bit unsigned values, compute their product and indicate whether or not an overflow occurred

x	y	x*y	overflow?
100	101		
010	011		
111	010		

Exercise 6: Binary Multiplication

- Given the following 3-bit unsigned values, compute their product and indicate whether or not an overflow occurred

x	y	x*y	overflow?
100	101	100	yes
010	011	110	no
111	010	110	yes

Multiplying with Shifts

- Multiplication is slow
- Bit shifting is kind of like multiplication, and is often faster
  - x * 8 = x << 3
  - x * 10 = x << 3 + x << 1

- Most compilers will automatically replace multiplications with shifts where possible

Exercise 7: Feedback

1. Rate how well you think this recorded lecture worked
   1. Better than an in-person class
   2. About as well as an in-person class
   3. Less well than an in-person class, but you still learned something
   4. Total waste of time, you didn't learn anything

2. How much time did you spend on this video lecture (including time spent on exercises)?

3. Do you have any comments or feedback?