

CFGs – TAKE 2

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## Admin

Lab/neural net package?

Assignment 6

## Grammars

Language view:

A grammar is a set of structural rules that govern the composition of sentences, phrases and words.

Computational view:

A grammar (often called a “formal grammar”) is a set of rules that describe what strings are valid in a formal language.

## CFG production rules

$$S \rightarrow NP VP$$

left hand side  
(single symbol)

right hand side  
(one or more symbols)

## CFG example

$S \rightarrow A B C$   
 $A \rightarrow I$   
 $B \rightarrow \text{really}$   
 $B \rightarrow \text{really, B}$   
 $C \rightarrow \text{like cs}$

## CFGs formally

$G = (\mathbf{NT}, \mathbf{T}, \mathbf{P}, \mathbf{S})$

**NT**: finite set of nonterminal symbols

**T**: finite set of terminal symbols, **NT** and **T** are disjoint

**P**: finite set of productions of the form  
 $A \rightarrow \alpha, A \in \mathbf{NT} \text{ and } \alpha \in (\mathbf{T} \cup \mathbf{NT})^*$

**S**  $\in$  **NT**: start symbol

## CFG example

Grammars "generate" or "derive" strings:

$S \rightarrow A B C$   
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S

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We can apply a rule by substituting the symbol on the left hand side with the symbols on the right

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I really like cs

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No more rules apply, so we're done!

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We can apply a rule by substituting the symbol on the left hand side with the symbols on the right

### CFG example

Grammars describe a language, i.e. the strings (aka sentences) that are part of that language

$S \rightarrow A B C$   
 $A \rightarrow I$   
 $B \rightarrow \text{really}$   
 $B \rightarrow \text{really, B}$   
 $C \rightarrow \text{like cs}$

I really, really, ... like cs

What language does this represent?

$S \rightarrow aS$   
 $S \rightarrow E$   
 $E \rightarrow bE$   
 $E \rightarrow b$

What language does this represent?

$S \rightarrow aS$   
 $S \rightarrow E$  } Two options      S  
 $E \rightarrow bE$   
 $E \rightarrow b$

What language does this represent?

$S \rightarrow aS$   
 $S \rightarrow E$   
 $E \rightarrow bE$   
 $E \rightarrow b$

S  
↓  
aS

What language does this represent?

$S \rightarrow aS$   
 $S \rightarrow E$   
 $E \rightarrow bE$   
 $E \rightarrow b$

aS  
↓  
aaS

What language does this represent?

$S \rightarrow aS$   
 $S \rightarrow E$   
 $E \rightarrow bE$   
 $E \rightarrow b$

$aaS$   
 $\downarrow$   
 $aaaS$

- Can do this as many times as we want  
 - Keeps adding more a's to the front

What language does this represent?

$S \rightarrow aS$   
 $S \rightarrow E$   
 $E \rightarrow bE$   
 $E \rightarrow b$

$aaaS$   
 $\downarrow$   
 $aaaE$

Eventually, apply second rule

What language does this represent?

$S \rightarrow aS$   
 $S \rightarrow E$   
 $E \rightarrow bE$   
 $E \rightarrow b$

Two options

$aaaE$

What language does this represent?

$S \rightarrow aS$   
 $S \rightarrow E$   
 $E \rightarrow bE$   
 $E \rightarrow b$

$aaaE$   
 $\downarrow$   
 $aaabE$

What language does this represent?

$S \rightarrow aS$   
 $S \rightarrow E$   
 $E \rightarrow bE$   
 $E \rightarrow b$

$aaabE$   
 $\downarrow$   
 $aaabbE$

What language does this represent?

$S \rightarrow aS$   
 $S \rightarrow E$   
 $E \rightarrow bE$   
 $E \rightarrow b$

$aaabbE$   
 $\downarrow$   
 $aaabbbE$

What language does this represent?

$S \rightarrow aS$   
 $S \rightarrow E$   
 $E \rightarrow bE$   
 $E \rightarrow b$

$aaabbE$   
 $\downarrow$   
 $aaabb...bE$

- Can do this as many times as we want  
 - Keeps adding more b's to the end

What language does this represent?

$S \rightarrow aS$   
 $S \rightarrow E$   
 $E \rightarrow bE$   
 $E \rightarrow b$

$aaabb...bE$   
 $\downarrow$   
 $aaabb...bb$

Eventually, apply second rule

### What language does this represent?

$S \rightarrow aS$   
 $S \rightarrow E$   
 $E \rightarrow bE$   
 $E \rightarrow b$

$aaabb\dots bE$   
 $\downarrow$   
 $aaabb\dots bb$

Grammar represents all strings with zero or more a's followed by one or more b's

### Notational convenience

$S \rightarrow aS$   
 $S \rightarrow E$   
 $E \rightarrow bE$   
 $E \rightarrow b$



$S \rightarrow aS \mid E$   
 $E \rightarrow bE \mid b$

### Often many ways to write the same language

$S \rightarrow aS \mid E$   
 $E \rightarrow bE \mid b$

$S \rightarrow aS \mid E$   
 $E \rightarrow Eb \mid b$

$S \rightarrow aS \mid aaS \mid E$   
 $E \rightarrow Eb \mid b$

### What languages do these represent?

$S \rightarrow aEa \mid bEb$   
 $E \rightarrow Ea \mid Eb \mid a \mid b$

$S \rightarrow aSb$   
 $S \rightarrow ab$

$S \rightarrow aaS \mid abS \mid baS \mid bbS \mid \epsilon$

nothing

### What languages do these represent?

$S \rightarrow aEa \mid bEb$       all strings of a's and b's that start  
 and end with the same letter  
 $E \rightarrow Ea \mid Eb \mid a \mid b$

$S \rightarrow aSb$       strings of a's followed by an  
 equal number of b's  
 $S \rightarrow ab$

$S \rightarrow aaS \mid abS \mid baS \mid bbS \mid \epsilon$   
 all strings of a's and b's with even length

### Writing CFGs

Write a CFG to represent the language containing all strings that start with a.

$S \rightarrow aT$   
 $T \rightarrow Ta \mid Tb \mid \epsilon$

### Writing CFGs

Write a CFG to represent the language containing all strings with exactly two b's.

$S \rightarrow TbTbT$   
 $T \rightarrow Ta \mid \epsilon$

### CFG: Another example

Many possible CFGs for English, here is an example (fragment):

$S \rightarrow NP VP$   
 $VP \rightarrow V NP$   
 $NP \rightarrow DetP N \mid DetP AdjP N$   
 $AdjP \rightarrow Adj \mid Adv AdjP$   
 $N \rightarrow boy \mid girl$   
 $V \rightarrow sees \mid likes$   
 $Adj \rightarrow big \mid small$   
 $Adv \rightarrow very$   
 $DetP \rightarrow a \mid the$

## Derivations in a CFG

$S \rightarrow NP VP$   
 $VP \rightarrow V NP$   
 $NP \rightarrow DetP N \mid DetP AdjP N$   
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S

What can we do?

## Derivations in a CFG

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NP VP

What can we do?

## Derivations in a CFG

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NP VP

## Derivations in a CFG

$S \rightarrow NP VP$   
 $VP \rightarrow V NP$   
 $NP \rightarrow \text{DetP } N \mid \text{DetP AdjP } N$   
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 $N \rightarrow \text{boy} \mid \text{girl}$   
 $V \rightarrow \text{sees} \mid \text{likes}$   
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DetP N VP

## Derivations in a CFG

$S \rightarrow NP VP$   
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 $NP \rightarrow \text{DetP } N \mid \text{DetP AdjP } N$   
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 $Adv \rightarrow \text{very}$   
 $DetP \rightarrow a \mid \text{the}$

the boy VP

## Derivations in a CFG

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 $VP \rightarrow V NP$   
 $NP \rightarrow \text{DetP } N \mid \text{DetP AdjP } N$   
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 $N \rightarrow \text{boy} \mid \text{girl}$   
 $V \rightarrow \text{sees} \mid \text{likes}$   
 $Adj \rightarrow \text{big} \mid \text{small}$   
 $Adv \rightarrow \text{very}$   
 $DetP \rightarrow a \mid \text{the}$

the boy likes NP

### Derivations in a CFG

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 $VP \rightarrow V NP$   
 $NP \rightarrow DetP N \mid DetP AdjP N$   
 $AdjP \rightarrow Adj \mid Adv AdjP$   
 $N \rightarrow boy \mid girl$   
 $V \rightarrow sees \mid likes$   
 $Adj \rightarrow big \mid small$   
 $Adv \rightarrow very$   
 $DetP \rightarrow a \mid the$

the boy likes a girl

### Derivations in a CFG: Order of Derivation Irrelevant

$S \rightarrow NP VP$   
 $VP \rightarrow V NP$   
 $NP \rightarrow DetP N \mid DetP AdjP N$   
 $AdjP \rightarrow Adj \mid Adv AdjP$   
 $N \rightarrow boy \mid girl$   
 $V \rightarrow sees \mid likes$   
 $Adj \rightarrow big \mid small$   
 $Adv \rightarrow very$   
 $DetP \rightarrow a \mid the$

the boy likes a girl

### Derivations of CFGs

Derivation history shows a tree:

the boy likes a girl

### Another CFG example

$S \rightarrow NP VP$   
 $VP \rightarrow V \mid V ADV$   
 $NP \rightarrow ART PreNP$   
 $PreNP \rightarrow N \mid ADJ PreNP$   
 $ADV \rightarrow furiously \mid soothingly \mid intentionally$   
 $ADJ \rightarrow colorless \mid green \mid smelly$   
 $ART \rightarrow the \mid a$   
 $V \rightarrow sleeps \mid eats \mid swims \mid sprints$   
 $N \rightarrow idea \mid bagel \mid milk \mid cow$

What can we generate?

### One last example

$S \rightarrow N$   
 $S \rightarrow ( S )$   
 $S \rightarrow S + S \mid S - S$   
 $S \rightarrow S * S \mid S / S$   
 $N \rightarrow 0 \mid 1 \mid 2 \mid \dots \mid 9$   
 $N \rightarrow NN$

What language does this CFG represent?

### One last example

$S \rightarrow N$   
 $S \rightarrow ( S )$   
 $S \rightarrow S + S \mid S - S$   
 $S \rightarrow S * S \mid S / S$   
 $N \rightarrow 0 \mid 1 \mid 2 \mid \dots \mid 9$   
 $N \rightarrow NN$

All arithmetic expressions!

### Parsing

Given a CFG and a sentence, determine the possible parse tree(s)

$S \rightarrow NP VP$   
 $NP \rightarrow N$   
 $NP \rightarrow PRP$   
 $NP \rightarrow N PP$   
 $VP \rightarrow V NP$   
 $VP \rightarrow V NP PP$   
 $PP \rightarrow IN N$   
 $PRP \rightarrow I$   
 $V \rightarrow eat$   
 $N \rightarrow sushi$   
 $N \rightarrow tuna$   
 $IN \rightarrow with$

I eat sushi with tuna

What parse trees are possible for this sentence?

How did you do it?

What if the grammar is much larger?

### Parsing

$S \rightarrow NP VP$   
 $NP \rightarrow PRP$   
 $NP \rightarrow N PP$   
 $NP \rightarrow N$   
 $VP \rightarrow V NP$   
 $VP \rightarrow V NP PP$   
 $PP \rightarrow IN N$   
 $PRP \rightarrow I$   
 $V \rightarrow eat$   
 $N \rightarrow sushi$   
 $N \rightarrow tuna$   
 $IN \rightarrow with$

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What is the difference between these parses?

