CS302 - Assignment 18

Due: Tuesday, May 1 at the beginning of class
Hand-in method: paper

1. **[2 points]** Determine wether the following statement is true or false. If false, give a counterexample. If true, give a brief (but concrete) explanation justifying the statement.

   Given a flow network G, let (L, R) be a minimum capacity cut in the flow graph. If we increase the capacity of all of the edges in the graph by 1, then (L, R) is still a minimum capacity cut in this new graph.

2. **[6 points]** VT has another flood and your in charge of routing people to hospitals. There are n people with injuries and there are k hospitals scattered throughout VT. Each of these people needs to be brought to a hospital within the next 30 minutes.

   So as not to overload any hospital, you'd like to distribute these people evenly across the hospitals, that is each hospital should get n/k people (you may assume that n divides equally into k).

   - Describe an algorithm that determines if this is possible. Make sure to state clearly what your checking for to determine if it is possible.
   - Very briefly, justify that your approach is correct (in particular, how you've handled all of the problem constraints).
   - If it is possible, describe how to determine where to send each of the n people.

3. **[6 points]** As we've seen in class, network flow is useful for matching in bipartite graphs. A *perfect matching* is a matching where every vertex in the bipartite graph has a match. Prove that the following holds: a perfect matching exists if and only if any subset of S nodes on one side of the bipartite graph is connected to at least $|S|$ nodes on the other side of the bipartite graph.

   Remember to prove an iff statement you must prove both directions. For example, to prove "A iff B", first, assume A is true and prove that B is therefore true and, second, assume B is true and prove that A is therefore true.

   Hint: Think about max-flow min-cut.

4. **[3 points]** To assign freshmen seminars to students, Prof. Scharstein sets up the task as a flow problem. There are n freshmen being assigned to m freshmen seminars, each of which

represent a node in the flow network. An edge is added from s to each freshmen node with capacity 1 and an edge is added with capacity 10 from each class node to the sink (representing a limit on class size of 10).

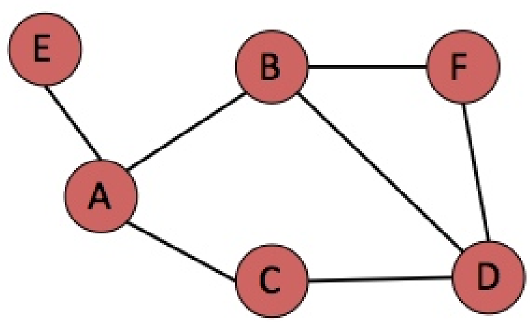
Each freshmen submits his/her top 5 choices for freshmen seminar classes. Each of these choices are entered into the flow network as unit capacity edges between the freshmen node and the 5 corresponding class nodes. The max-flow is then determined and assuming the flow=n, then each student is assigned to the course corresponding to where the node's one flow unit travels.

You're a freshmen and you find out that this is how freshmen seminar classes are chosen. There is one class that you really, really want to get in to. You manage to obtain the ranking of last year's freshmen seminars by popularity.

Given that you know the selection process is the one described above and assuming that this year's offerings are the same as last year's (and the interest level will be similar this year), how should you select your five choices so as to increase your odds of getting your top choice? Justify your answer.

Extra Credit

5. **[2 points]** Given an undirected graph one common question to what is the smallest subset of vertices S such that for *every* edge in the graph includes one of those vertices, that is $u \in S$ or $v \in S$ for all edges (u, v). For example, given the graph



the smallest set of vertices that meets the constraints is: $\{D, A, F\}$. Notice there are other *non-optimal* answers such as $\{E, B, C, F\}$, but this contains more vertices so would not be correct.

For general graphs, this problem turns out to be hard. However, for bipartite graphs, this problem is more tractable.

Describe an algorithm, prove that it's correct and state the run-time that solves the above problem on a *bipartite graph.*

Hint: Draw a few examples by hand and work through them.