

CS159 - Absolute Discount Smoothing Handout

Given the following corpus (where we only have one letter words):

a a a b
a b b a
c a a a

We would like to calculate an absolute discounted model with $D = 0.5$. We'll ignore the begin and end sentence tokens as well and assume that our vocabulary is all three "words".

We first calculate the unigram MLE probabilities as:

	MLE prob
$p(a)$	8/12
$p(b)$	3/12
$p(c)$	1/12

and the bigram MLE probabilities as:

	MLE prob
$p(a a)$	4/6
$p(b a)$	2/6
$p(c a)$	0
$p(a b)$	1/2
$p(b b)$	1/2
$p(c b)$	0
$p(a c)$	1
$p(b c)$	0
$p(c c)$	0

Using this information, we can now calculate the reserved mass and the α s for each of the words in our vocabulary (i.e. things that we're conditioning on):

- a

The numerator for our α is the reserved mass:

$$\text{reserved_mass}(a) = (2 * 0.5)/6 = 1/6$$

and the denominator is:

$$1 - \sum_{x:\text{count}(ax)>0} p(x) = 1 - (p(a) + p(b)) = 1 - (8/12 + 3/12) = 1/12$$

giving us an α of:

$$\alpha(a) = \frac{1/6}{1/12} = 2$$

- b

$$\text{reserved_mass}(b) = (2 * 0.5)/2 = 1/2$$

$$1 - \sum_{x:\text{count}(bx)>0} p(x) = 1 - (p(a) + p(b)) = 1 - (8/12 + 3/12) = 1/12$$

$$\alpha(b) = \frac{1/2}{1/12} = 6$$

- c

$$\text{reserved_mass}(c) = (1 * 0.5)/1 = 1/2$$

$$1 - \sum_{x:\text{count}(cx)>0} p(x) = 1 - p(a) = 1 - (8/12) = 4/12 = 1/3$$

$$\alpha(c) = \frac{1/2}{1/3} = 3/2$$

Finally, now that we have the α s, we can calculate the smoothed bigram probabilities. For those that occurred, we simply discount the count. For those that did not occur, we calculate the probability as *alpha* times the unigram probability of the word.

	eqn	prob
$p(a a)$	$(4 - 0.5)/6$	$3.5/6$
$p(b a)$	$(2 - 0.5)/6$	$2.5/6$
$p(c a)$	$2 * 1/12$	$1/6$
$p(a b)$	$(1 - 0.5)/2$	$1/4$
$p(b b)$	$(1 - 0.5)/2$	$1/4$
$p(c b)$	$6 * 1/12$	$1/2$
$p(a c)$	$(1 - 0.5)/1$	$1/2$
$p(b c)$	$3/2 * 3/12$	$3/8$
$p(c c)$	$3/2 * 1/12$	$1/8$

Notice that after the smoothing, the three distributions all still sum to 1. In this case discounting by 0.5 may be a bit aggressive.