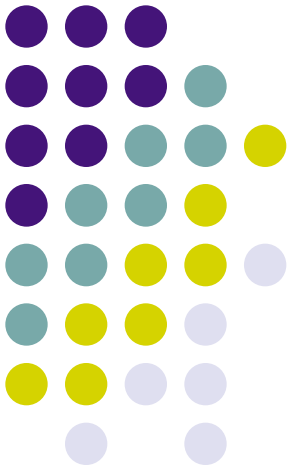
Quicksort

David Kauchak

cs062

Spring 2010

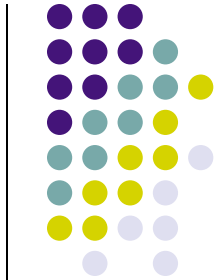


```
public static int partition(double[] nums, int start, int end){
   int lessThanIndex = start-1;

   for( int i = start; i < end; i++ ){
      if( nums[i] <= nums[end] ){
         lessThanIndex++;
         swap(nums, lessThanIndex, i);
      }
   }

   swap(nums, lessThanIndex+1, end);

   return lessThanIndex+1;
}
```



what does this method do?

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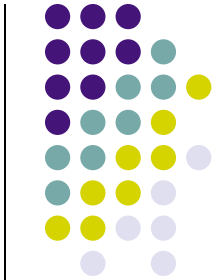
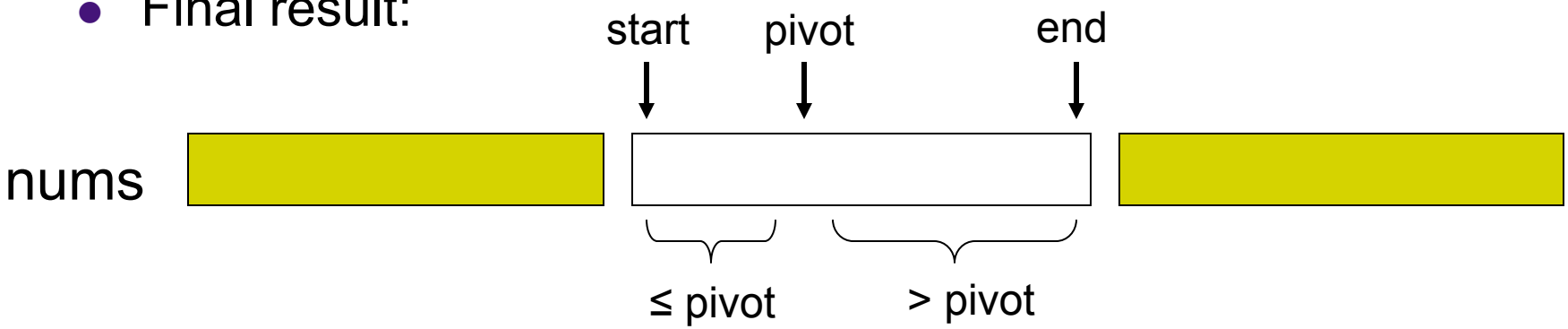
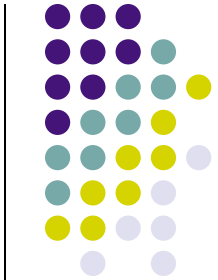
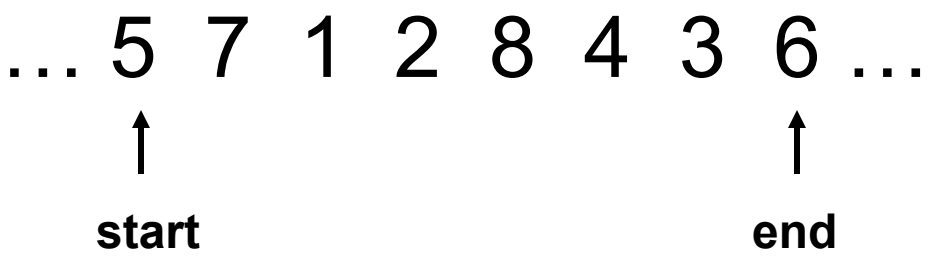


- nums[end] is called the ***pivot***
- Partitions the elements A[start…end-1] in to two sets, those ≤ pivot and those > pivot
- Operates in place
- Final result:





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      }
   }
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   return lessThanIndex+1;
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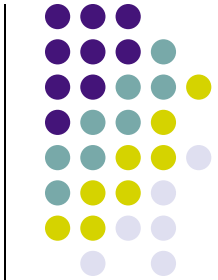

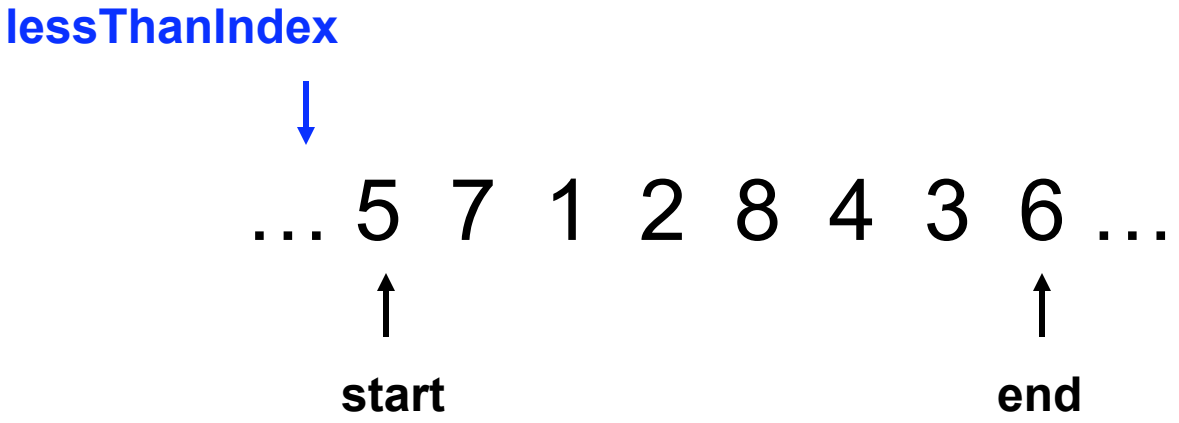


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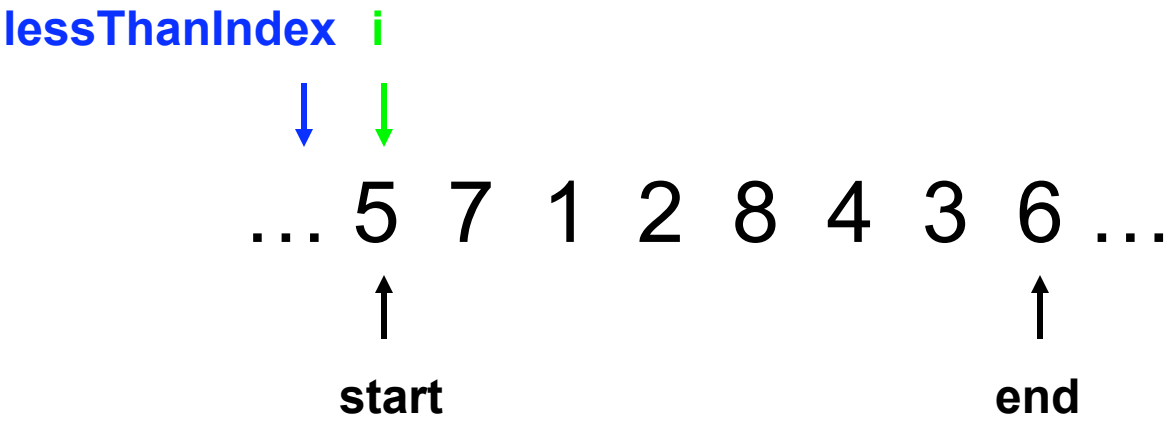

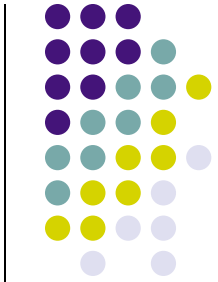

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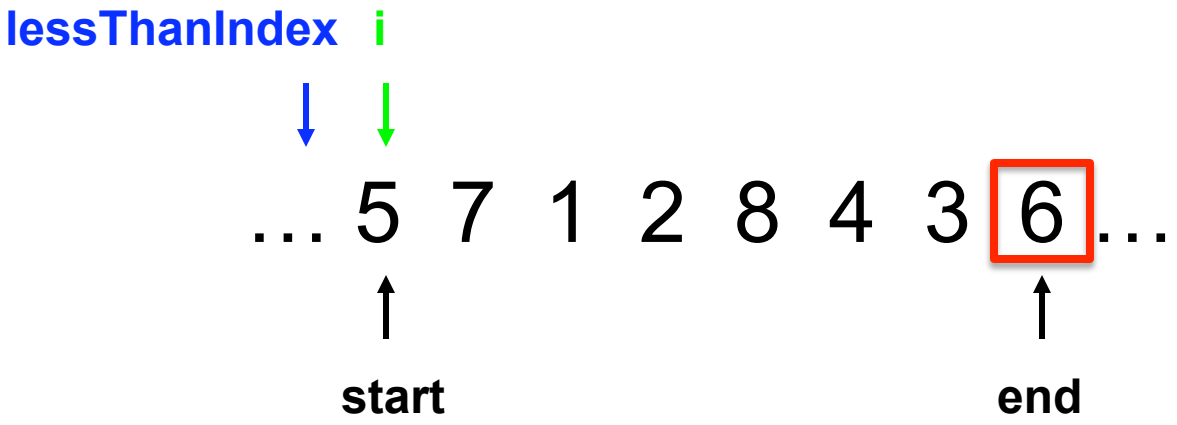


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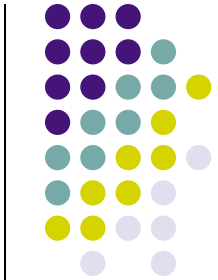

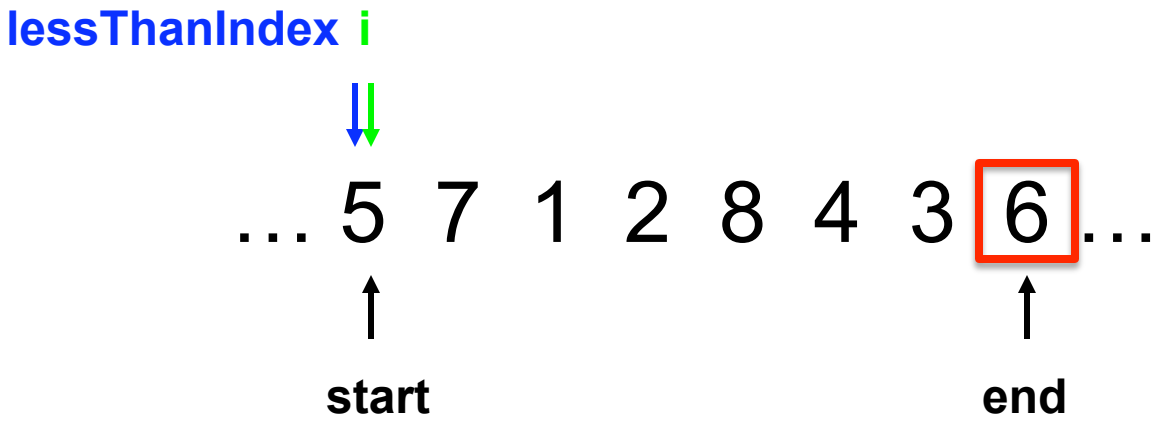


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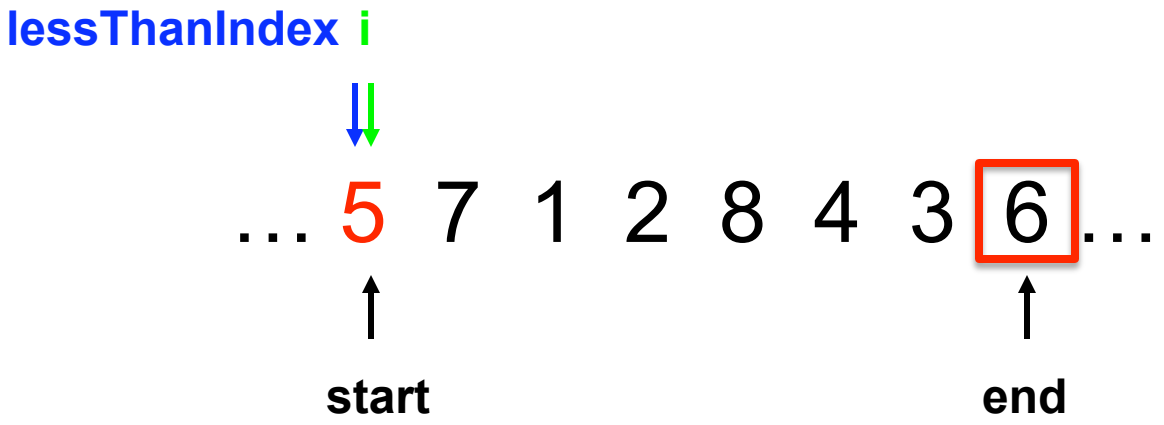


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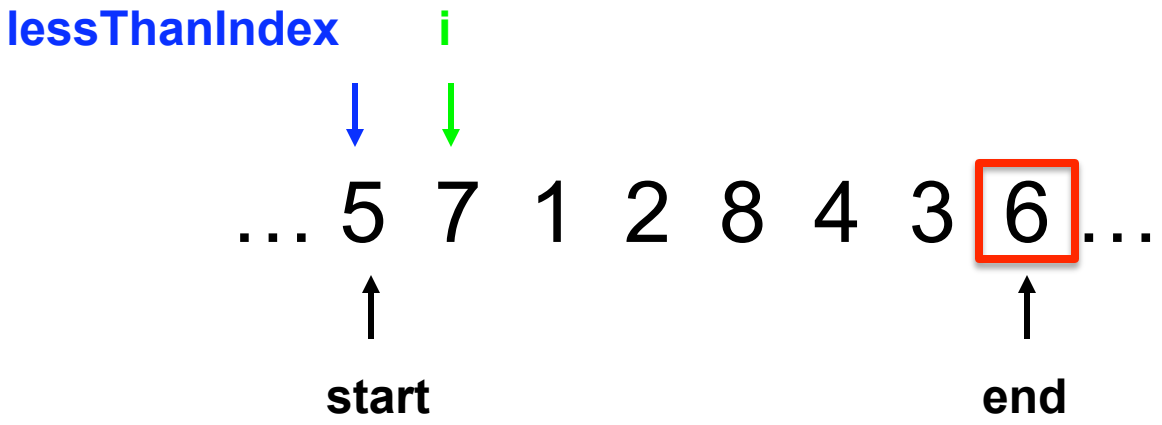


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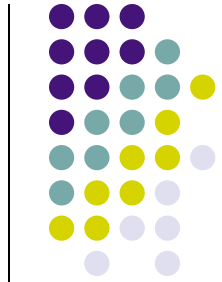

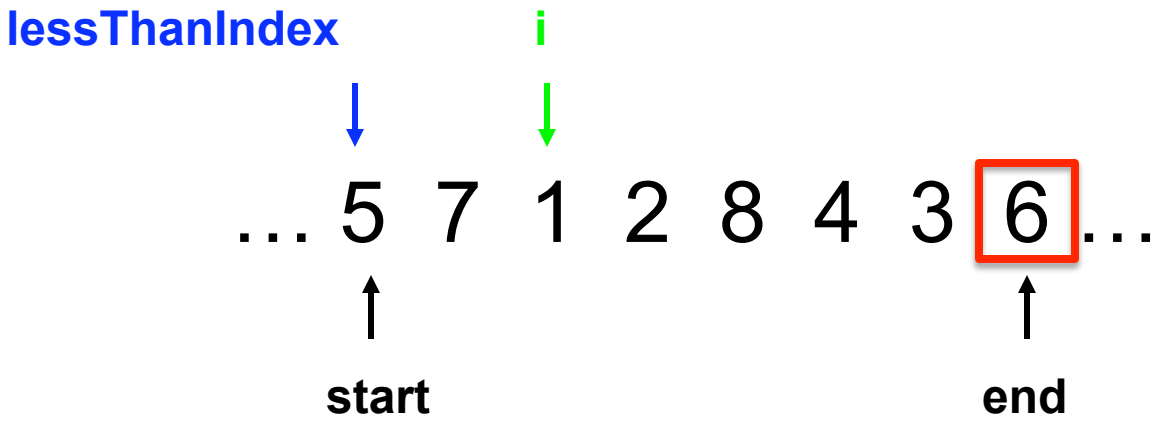


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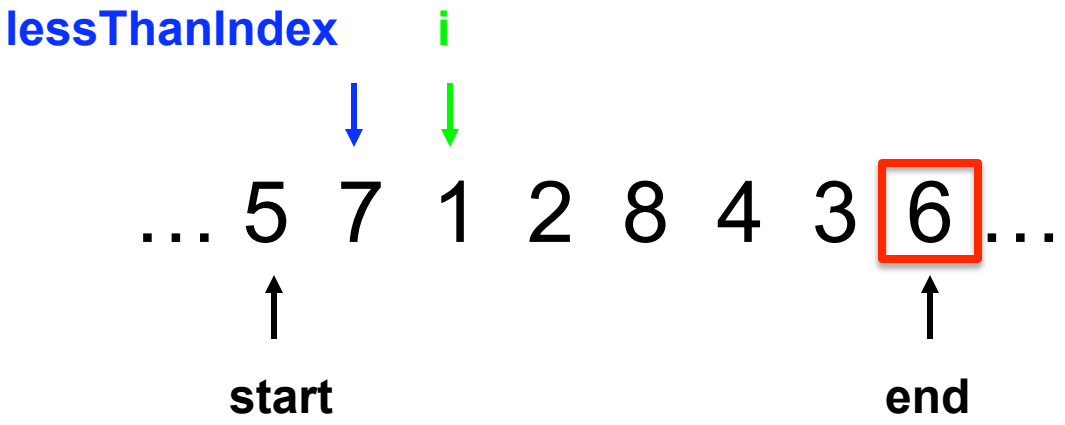

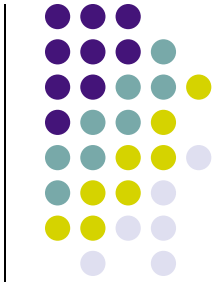

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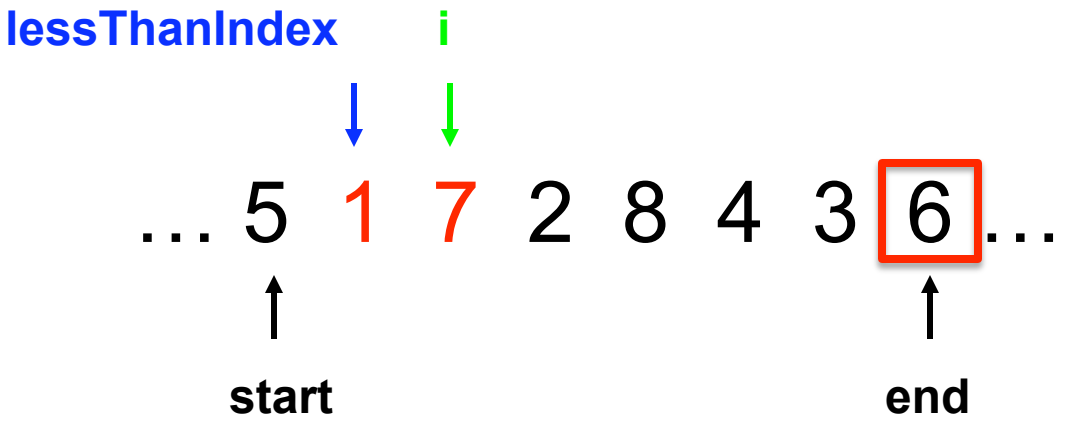


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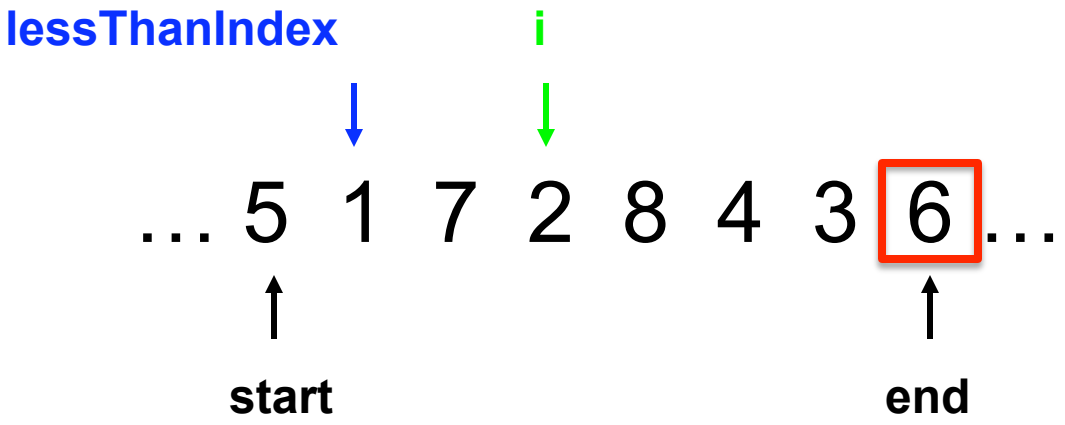

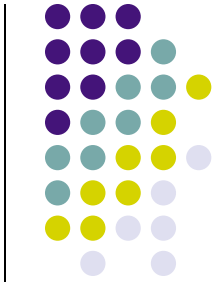

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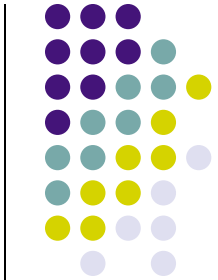

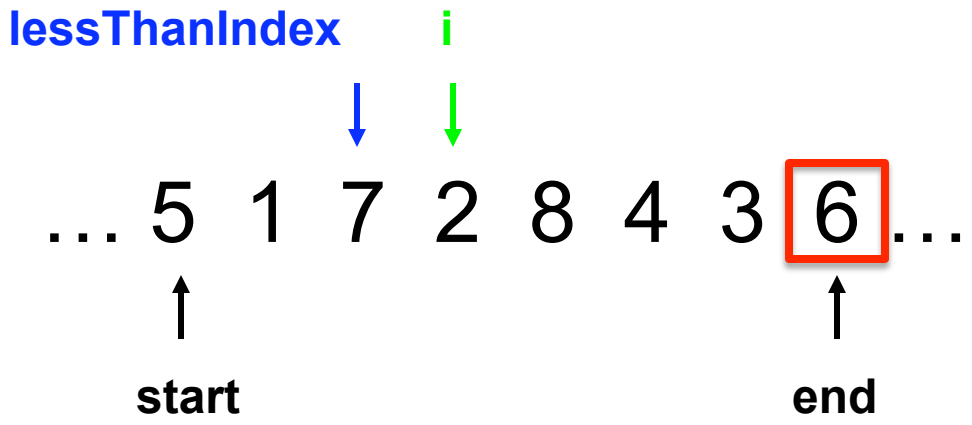


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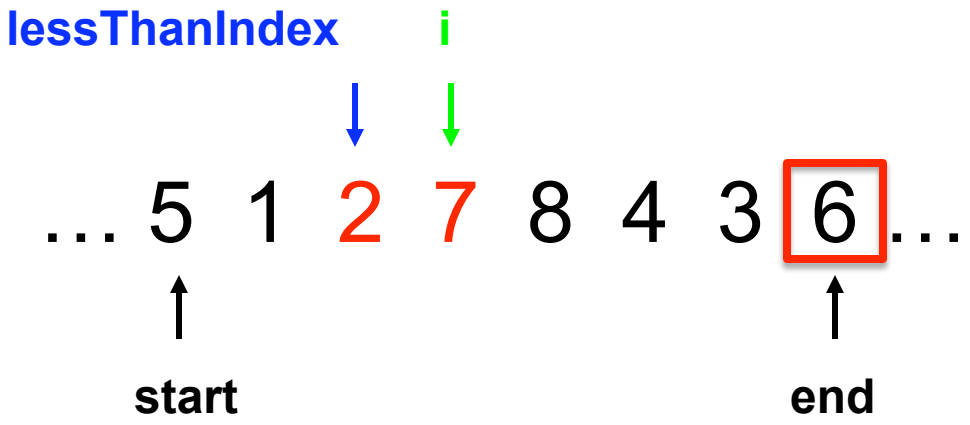


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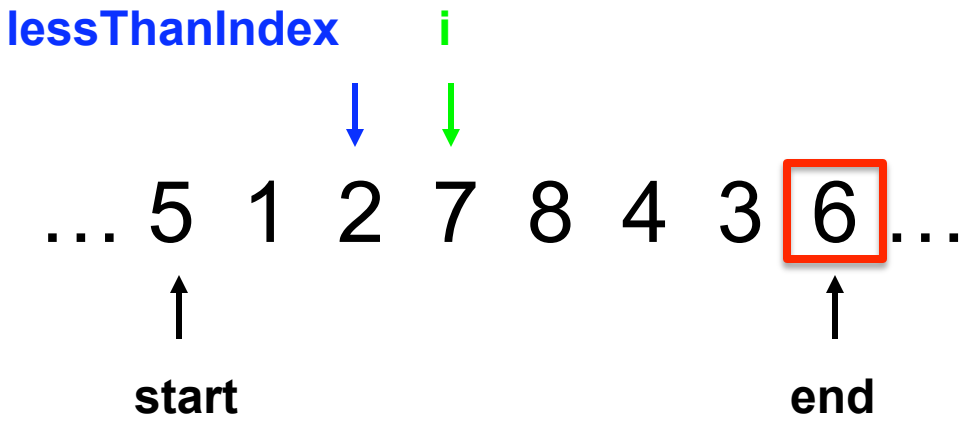


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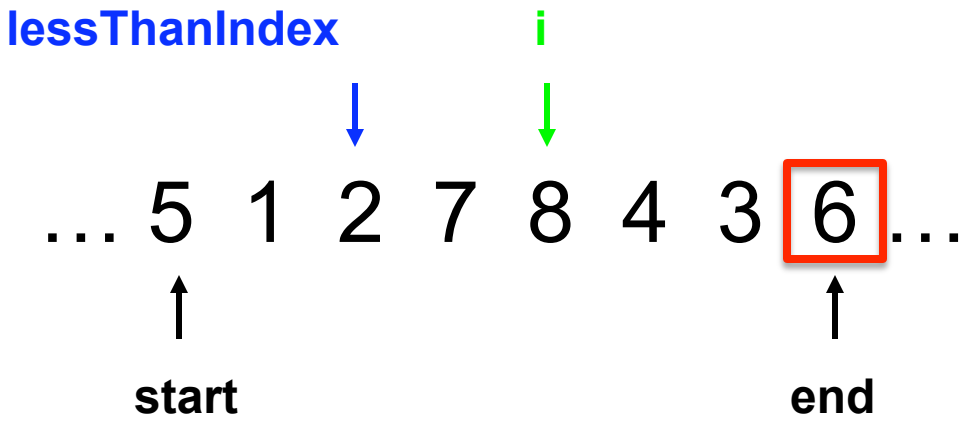


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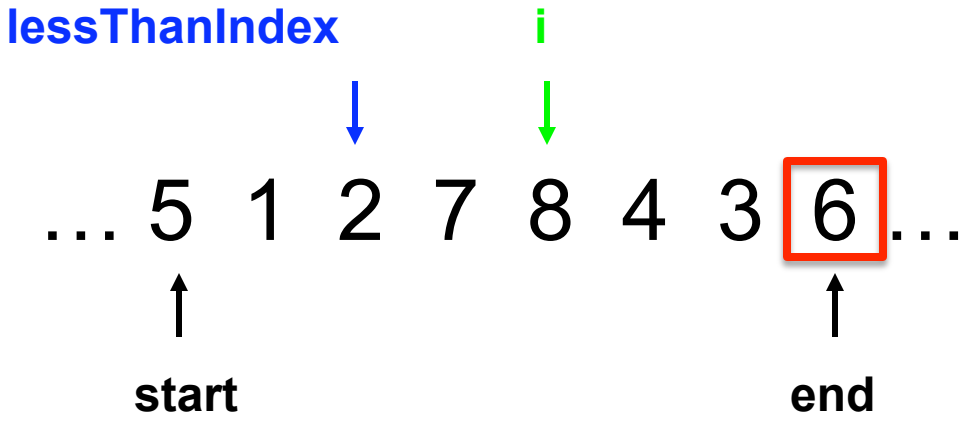
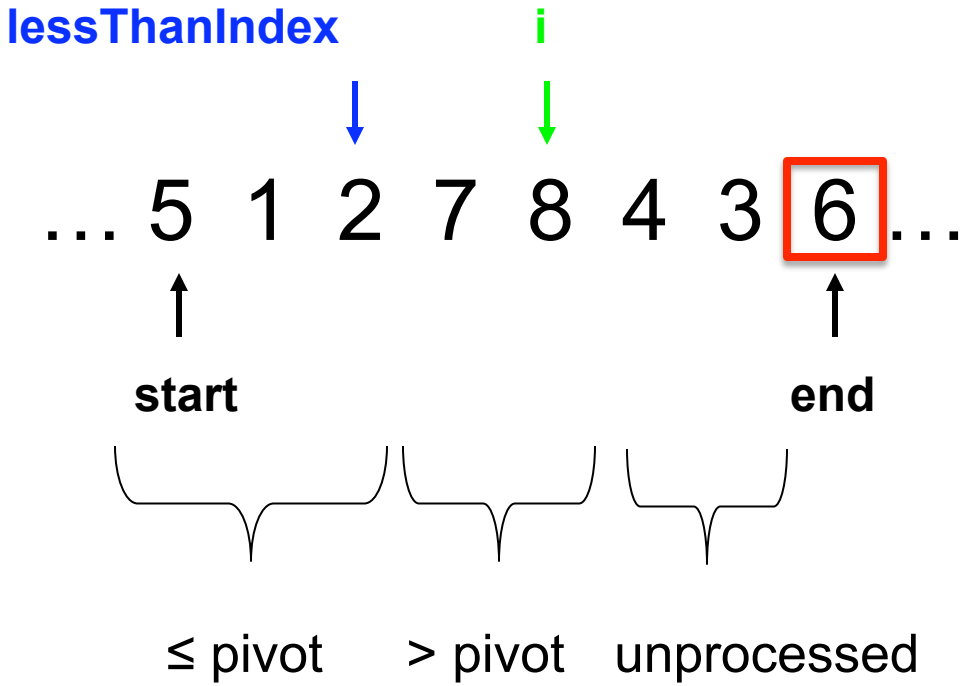
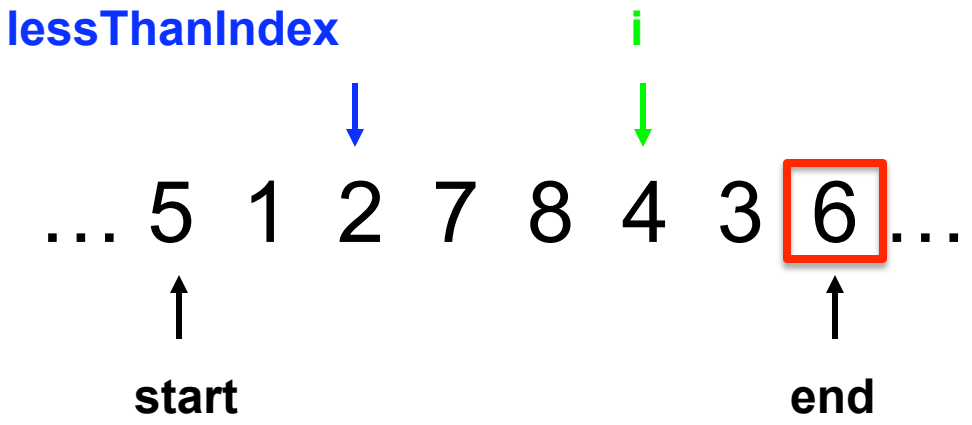


What's happening?


lessThanIndex
i
... 5 1 2 7 8 4 3 6 ...
start
end
≤ pivot
> pivot
unprocessed

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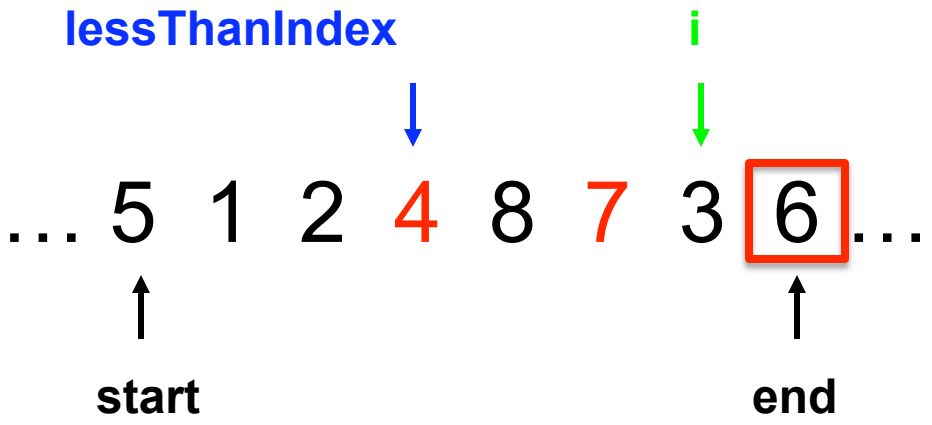


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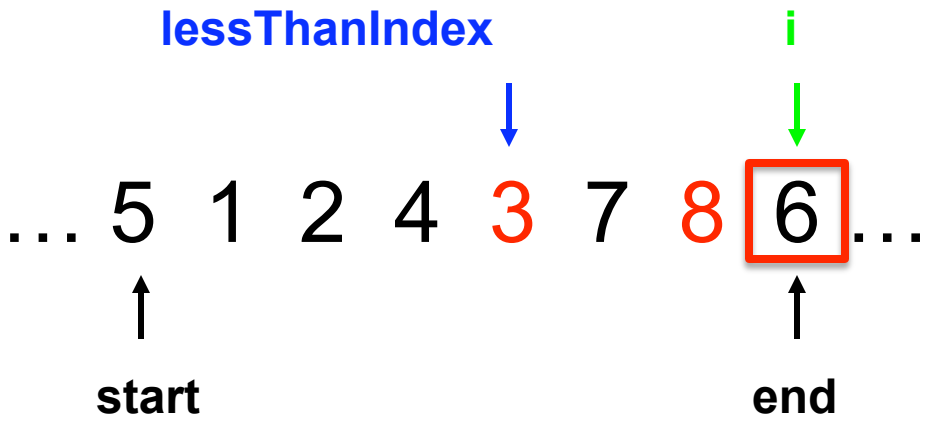


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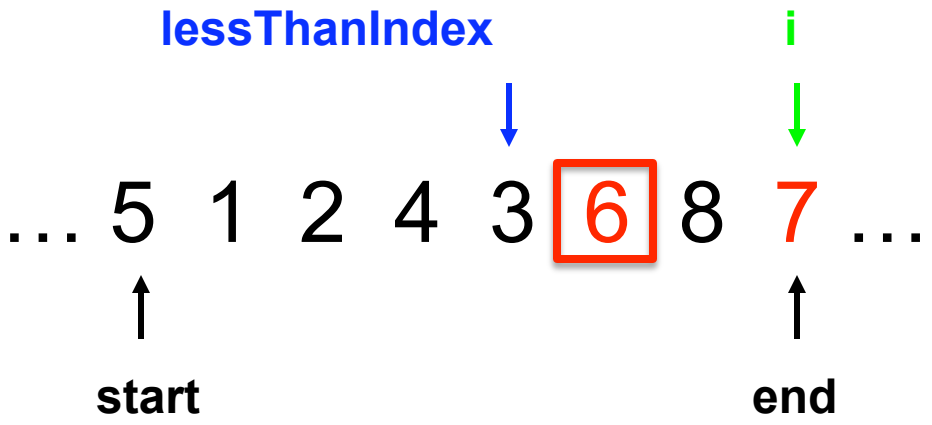


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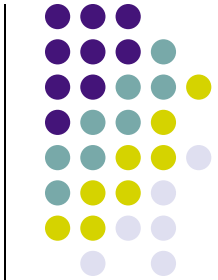
Partition running time?



- **O(n)**

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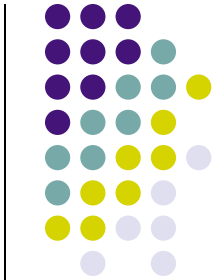
Quicksort



How can we use this method to sort nums?

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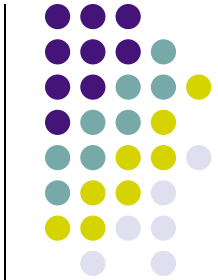
Quicksort



```
private static void quicksortHelper(double[] nums, int start, int end){
   if( start < end ){
      int partition = partition(nums, start, end);
      quicksortHelper(nums, start, partition-1);
      quicksortHelper(nums, partition+1, end);
   }
}

public static int partition(double[] nums, int start, int end){
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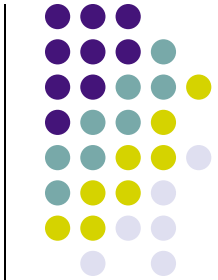

8	5	1	3	6	2	7	4

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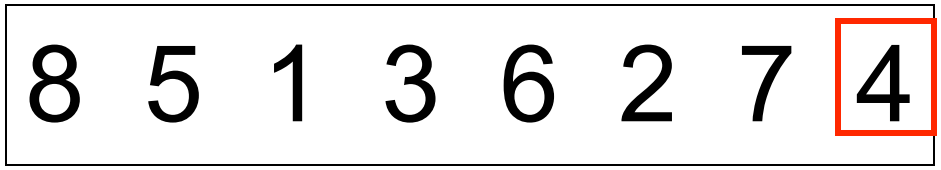


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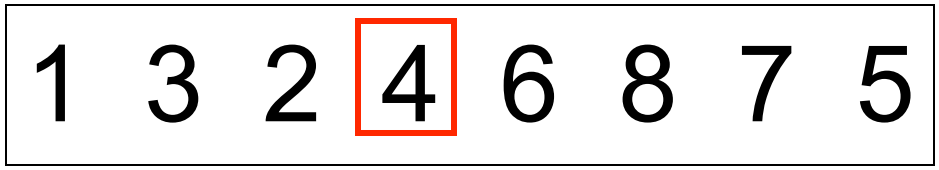


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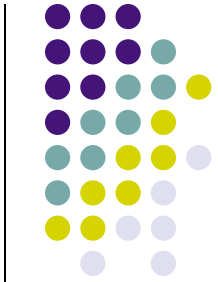

1 3 2 4 6 8 7 5

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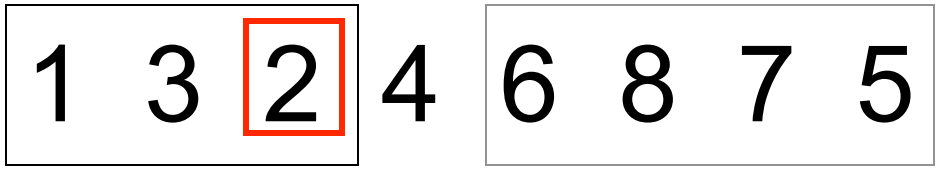


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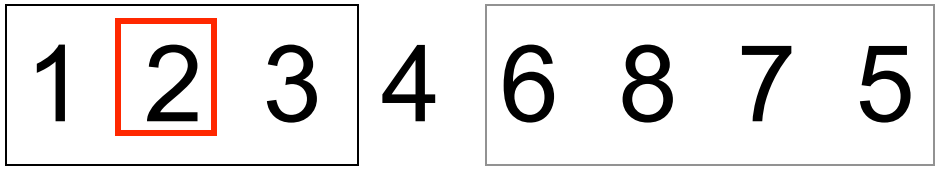


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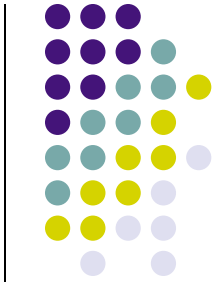

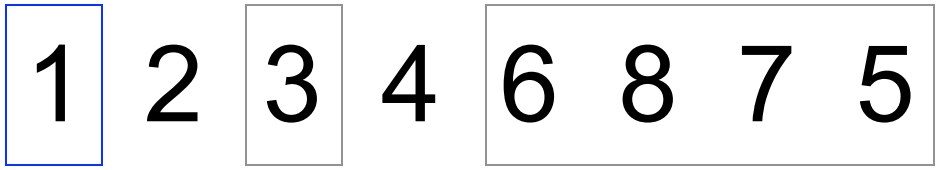


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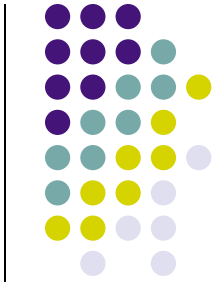

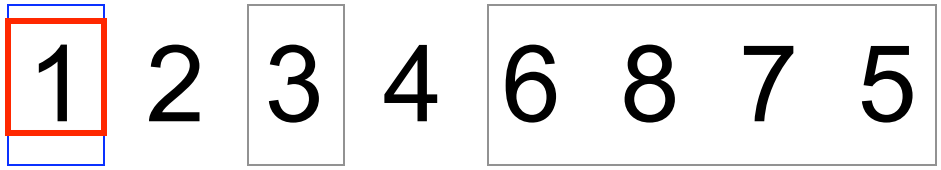


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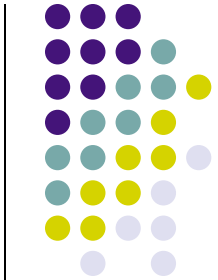

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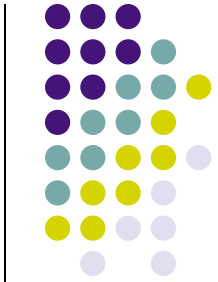

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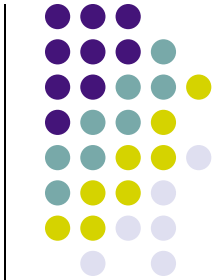

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What happens here?

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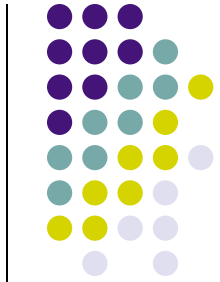

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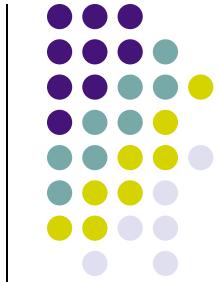

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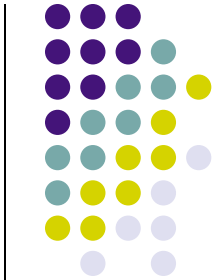

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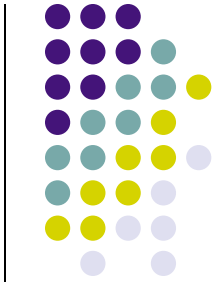

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   }
}
```
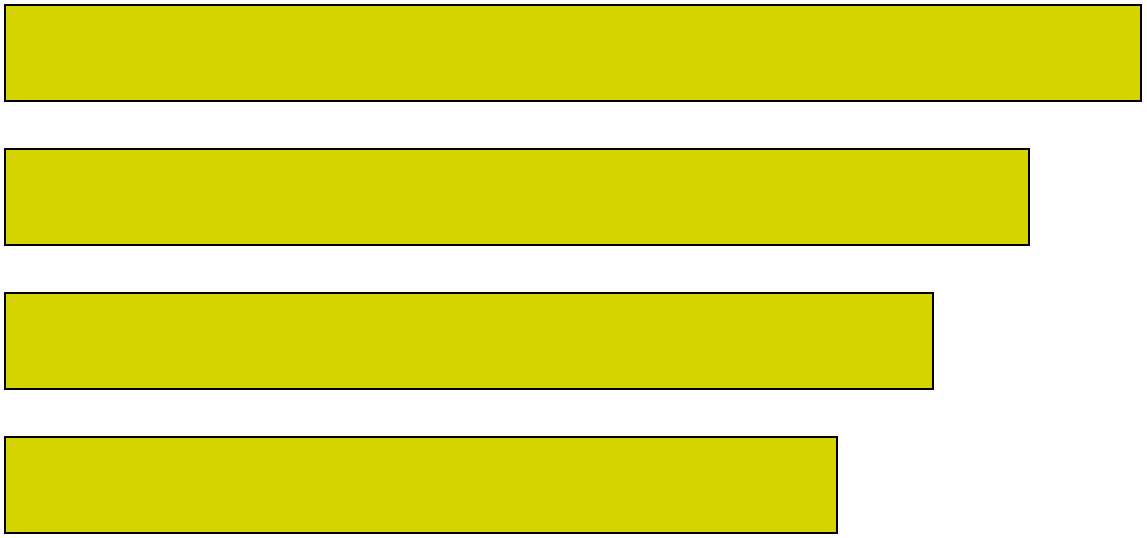

Running time of Quicksort?

- Worst case?
- Each call to Partition splits the array into an empty array and n-1 array



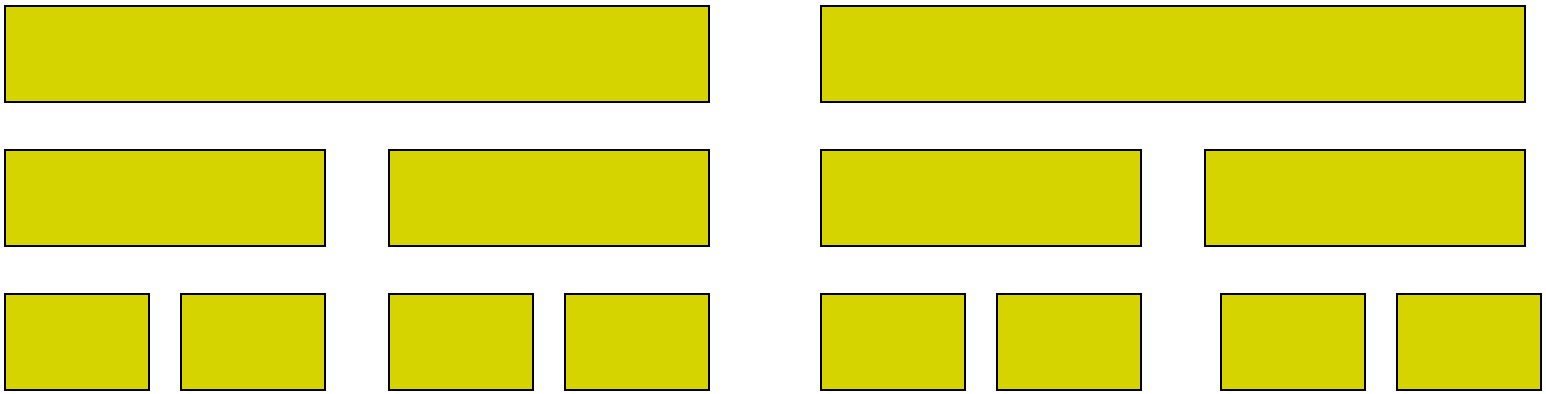
Quicksort: Worst case running time

$$n-1 + n-2 + n-3 + \ldots + 1 = O(n^2)$$

- When does this happen?
  - sorted
  - reverse sorted
  - near sorted/reverse sorted

Quicksort best case?

- Each call to Partition splits the array into two equal parts



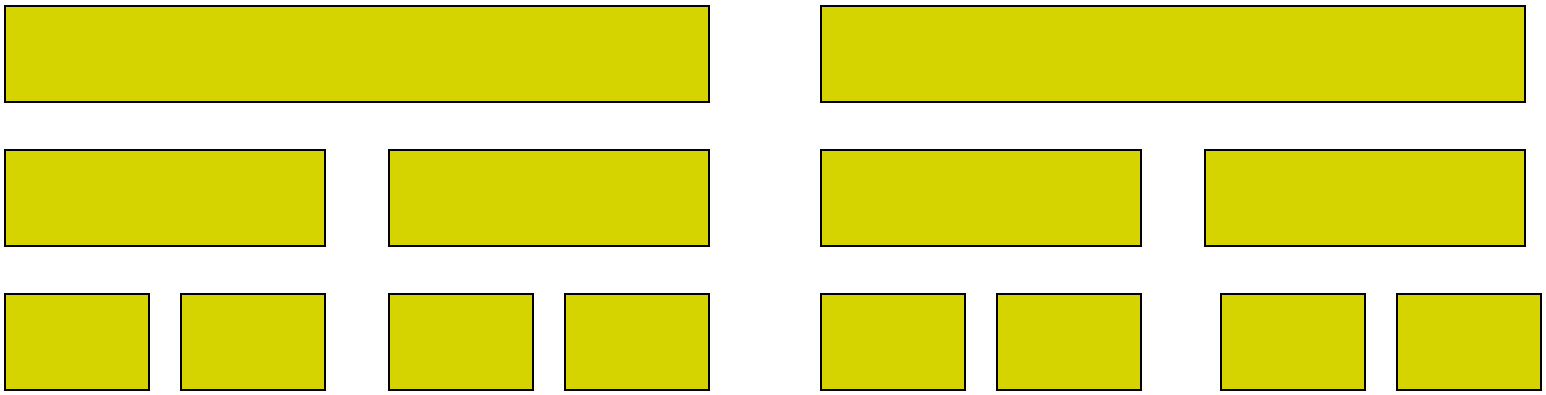
...

How much work is done at each “level”,
i.e. running time of a level?

O(n)

Quicksort best case?

- Each call to Partition splits the array into two equal parts



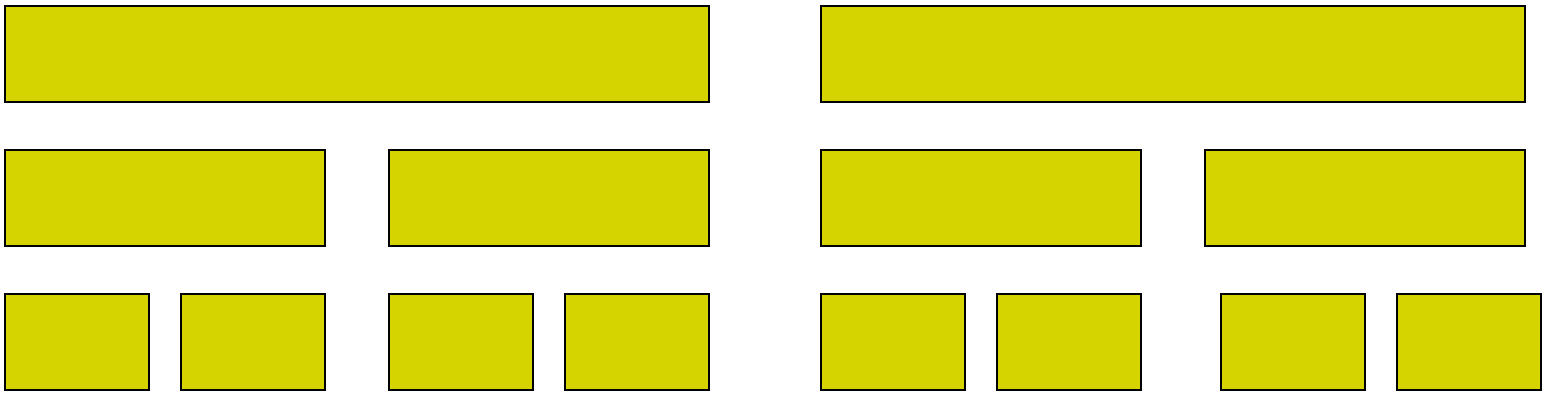
. . .

How many levels are there?

Similar to binary search, each call to Parition will throw away half the data until we're down to one element: $\log_2 n$ levels

Quicksort best case?

- Each call to Partition splits the array into two equal parts



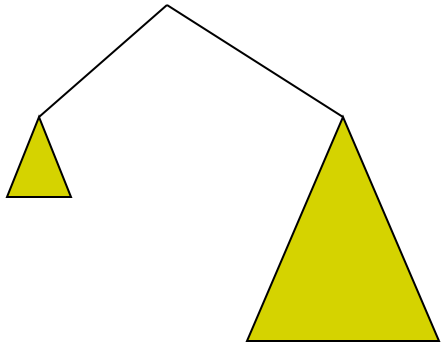
...

Overall runtime?

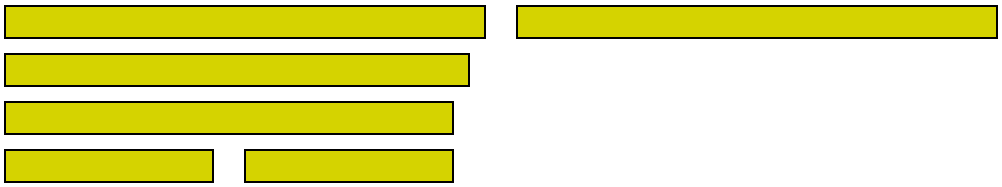
O(n log n)

Quicksort Average case?

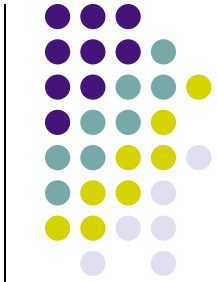
- Two intuitions
  - As long as the Partition procedure always splits the array into some constant ratio between the left and the right, say L-to-R, e.g. 9-to-1, then we maintain O(n log n)



  - As long as we only have a constant number of “bad” partitions intermixed with a “good partition” then we maintain O(n log n)



How can we avoid the worst case?

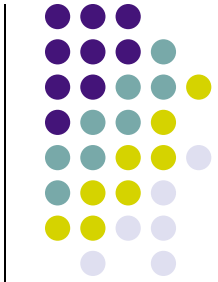


- Inject randomness into the data

```
private static void randomizedPartition(double[] nums, int start, int end){
   int i = random(start, end);
   swap(nums, i, end);
   return partition = partition(nums, start, end);
}
```

Randomized quicksort is average case O(n log n)

What is the wost case running time of randomized Quicksort?



$O(n^2)$

We could still get very unlucky and pick “bad” partitions at every step