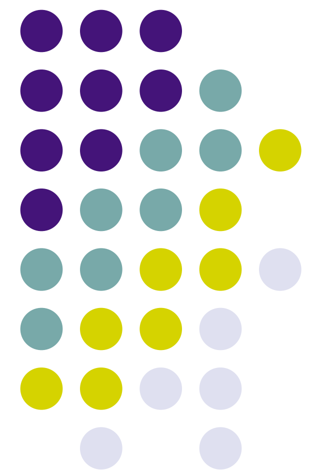
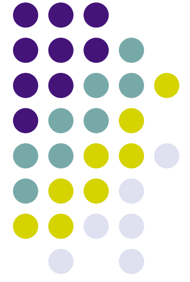
Dijkstra's Algorithm: single source shortest paths

David Kauchak

cs62

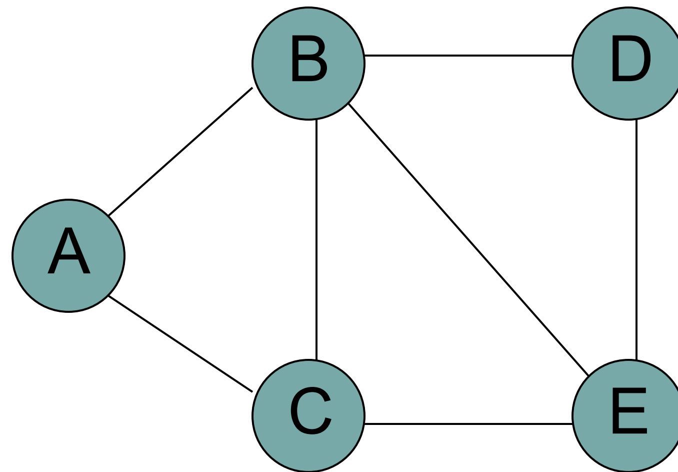
Spring 2010





Shortest paths

- What is the shortest path from a to d?



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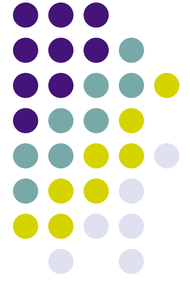
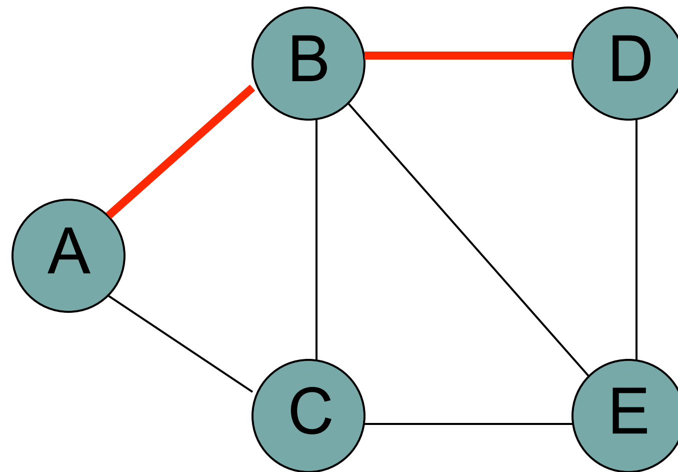
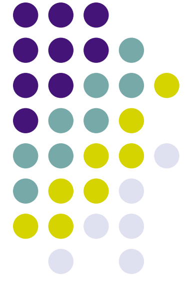


- BFS





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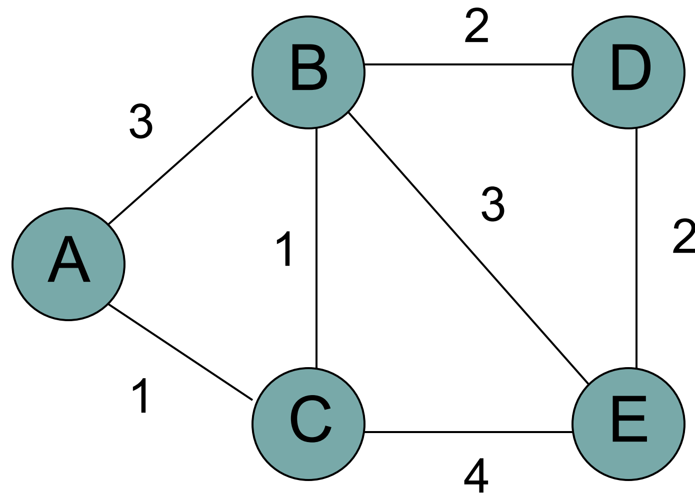

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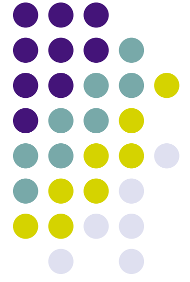

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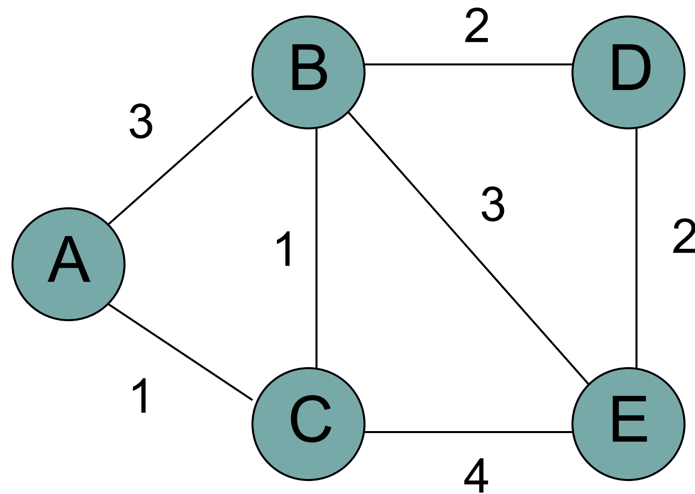

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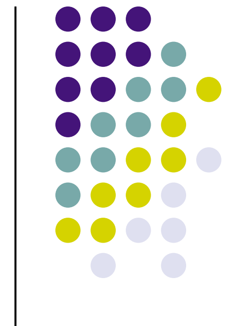

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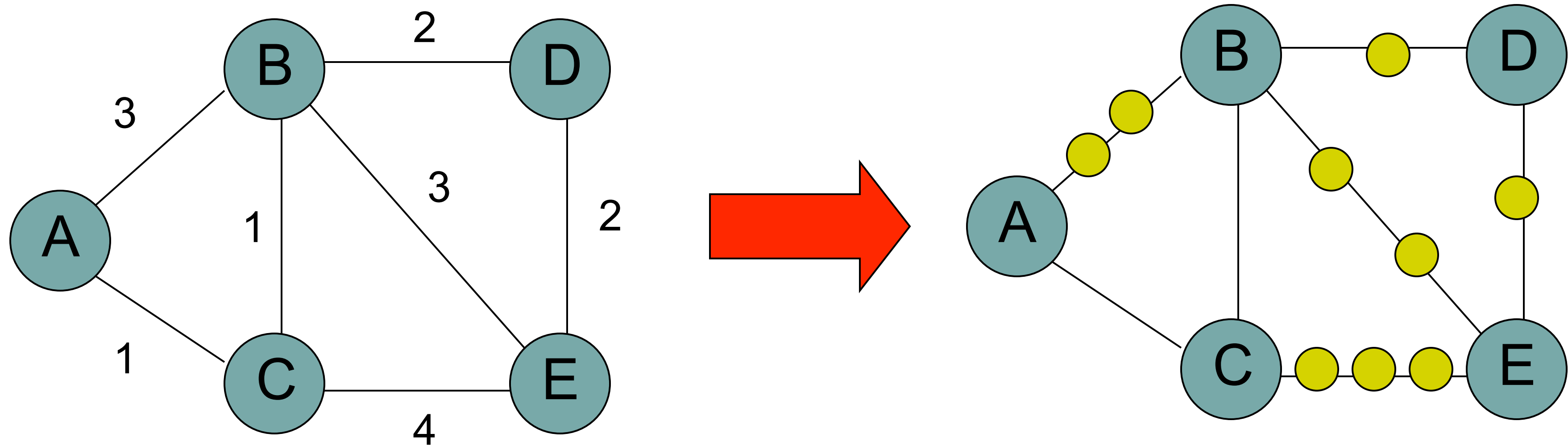

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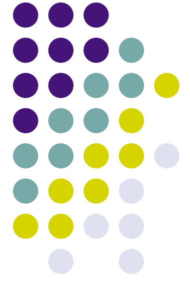

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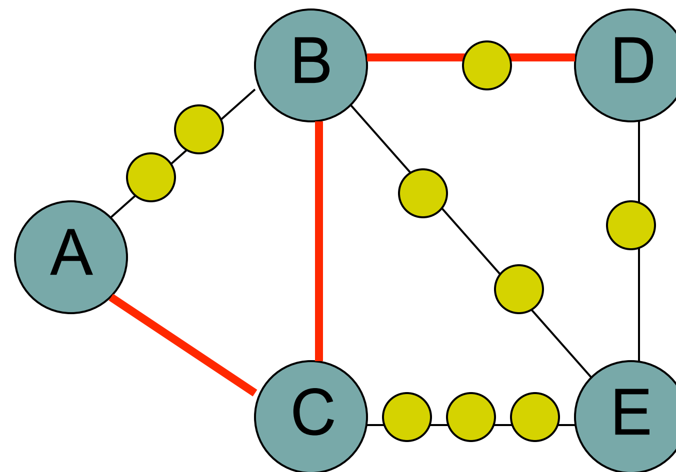
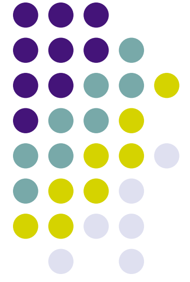
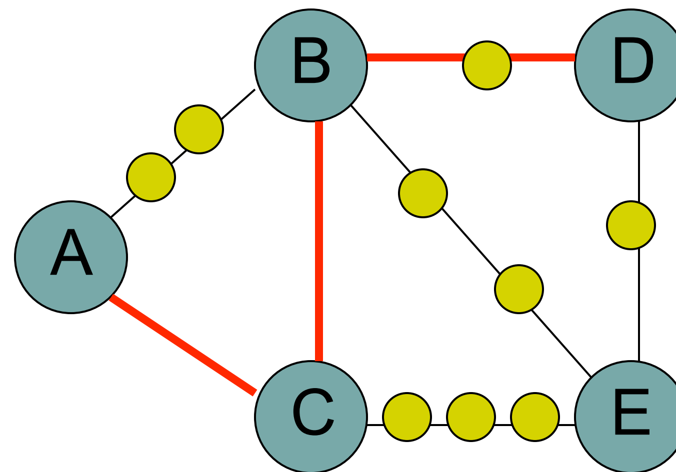
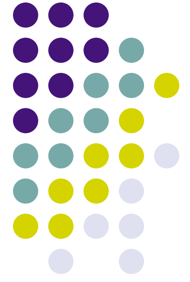




Shortest paths

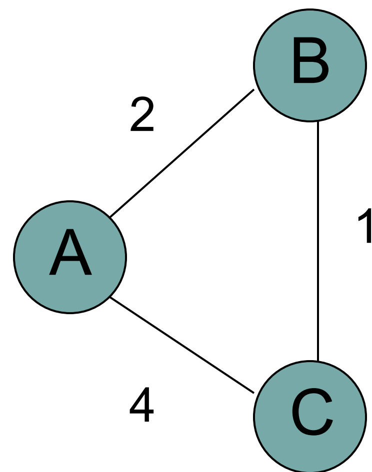
- What is the problem?

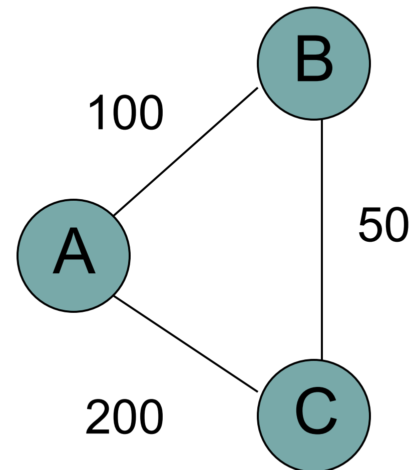


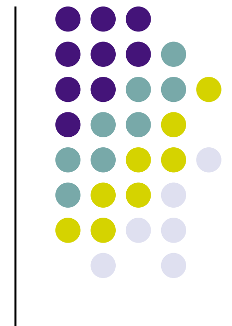


Shortest paths

- Running time is dependent on the weights







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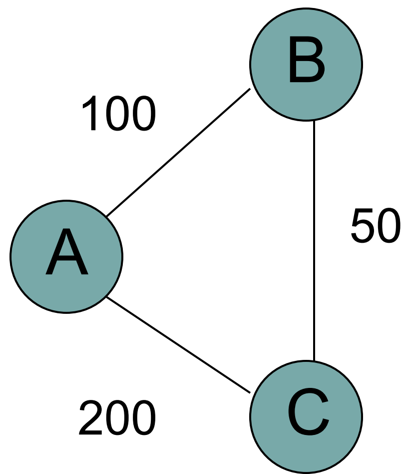

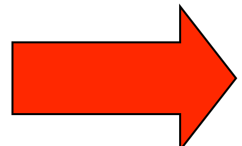

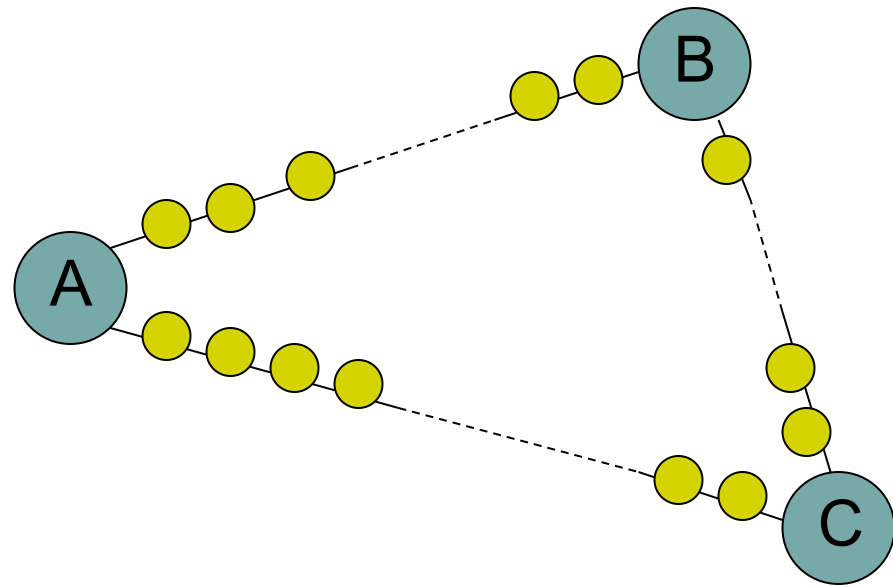

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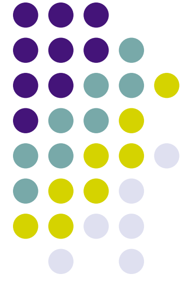

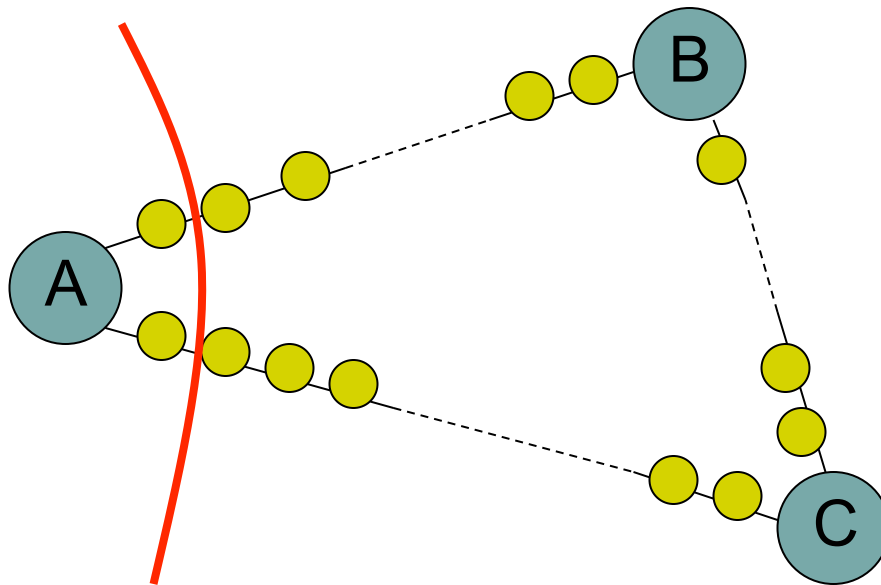

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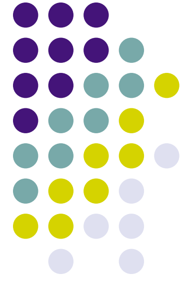

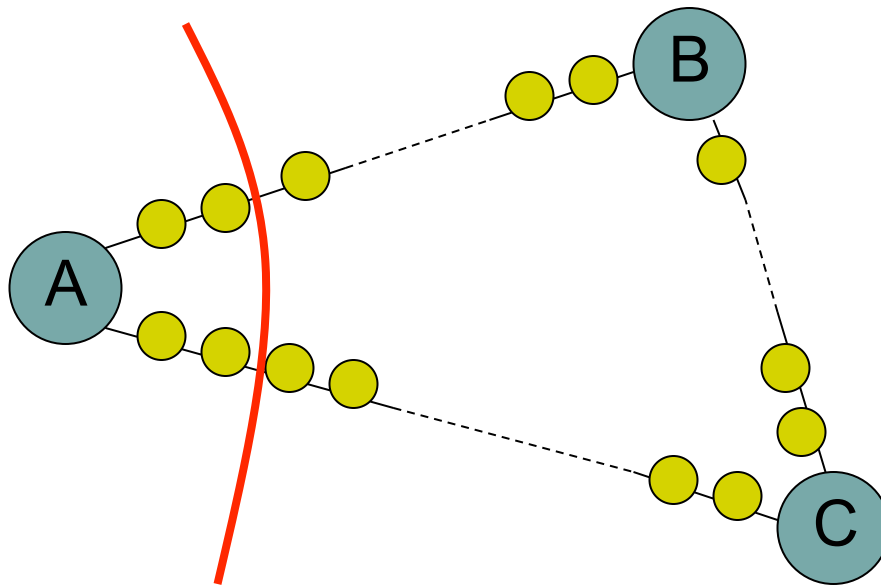

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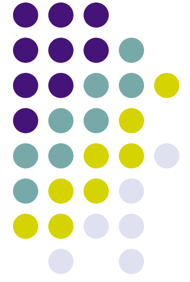
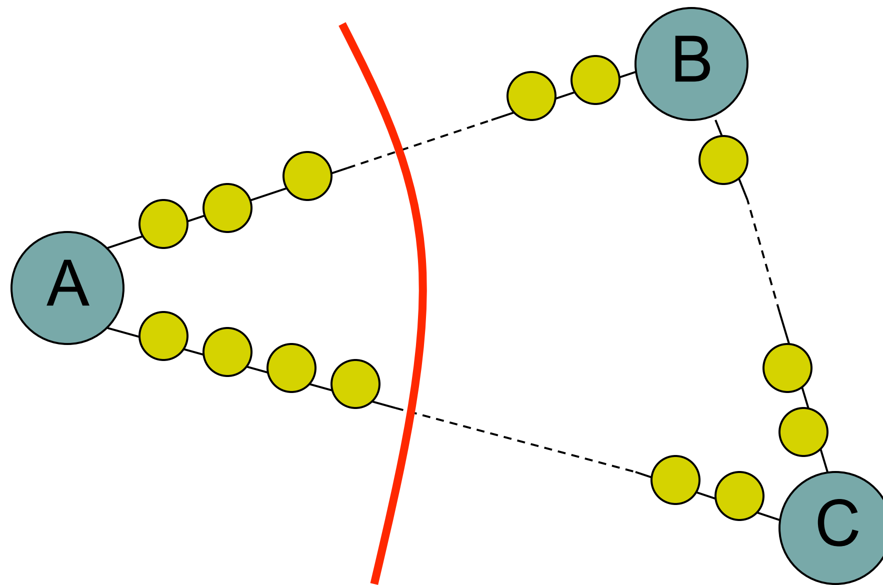
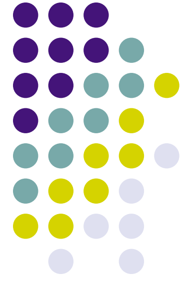


- Nothing will change as we expand the frontier until we've gone out 100 levels



Dijkstra’s algorithm



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                const map<int,list<pair<int,int> > > & graph) {
  map<int,int> parents;
  priorityqueue62 frontier;

  parents[start]=start;
  frontier.push(start, 0);

  while (!frontier.is_empty()) {
    int v = frontier.top_serialnumber();
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    frontier.pop();

    for (the neighbors (n,w) of v)
      if (n == parents[v])
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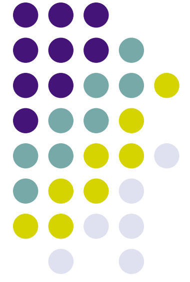
      else if (n is not in the frontier and has not been visited){
        parents[n] = v;
        frontier.push(n, p + w);
      }else if (p + w < frontier.get_priority(n)) {
        parents[n] = v;
        frontier.reduce_priority(n, p + w);
      }
    } // end while
  return parents;
}
```

Uses a priority queue to keep track of the next shortest path from the starting vertex

Vertices are kept in three sets:

- “visited”: those vertices who’s correct paths have been found. This occurs when a vertex is popped off the queue
- “frontier”: those vertices that we know about and have **a** path for, but not necessarily the vertices’ shortest paths. Vertices on the frontier are in the queue
- “rest”: the remaining vertices that we have not seen yet

Dijkstra’s algorithm



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BFS

```
enqueue start;
while (queue not empty) {
  dequeue v;

  if (v is not visited) {
    visit v;
    enqueue all of v’s neighbors;
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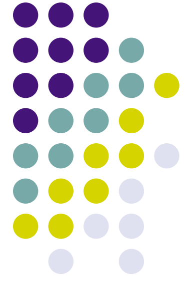

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- “parents” keeps track of shortest path
- only keep track of what the next vertex on the shortest path is

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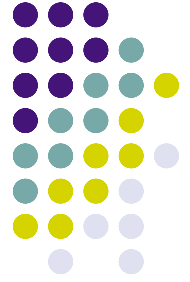

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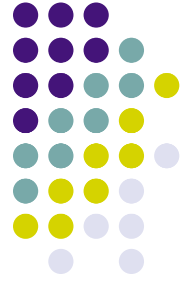

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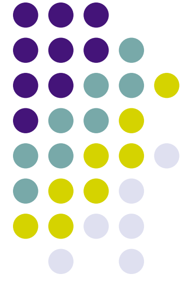

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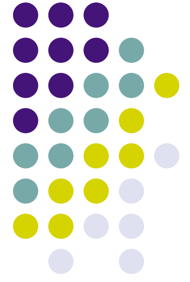

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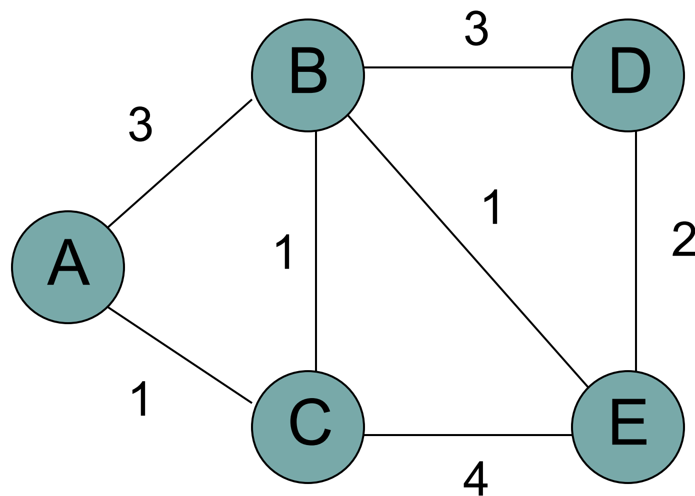

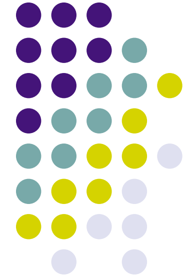

Heap	Parent
A 0	A: A

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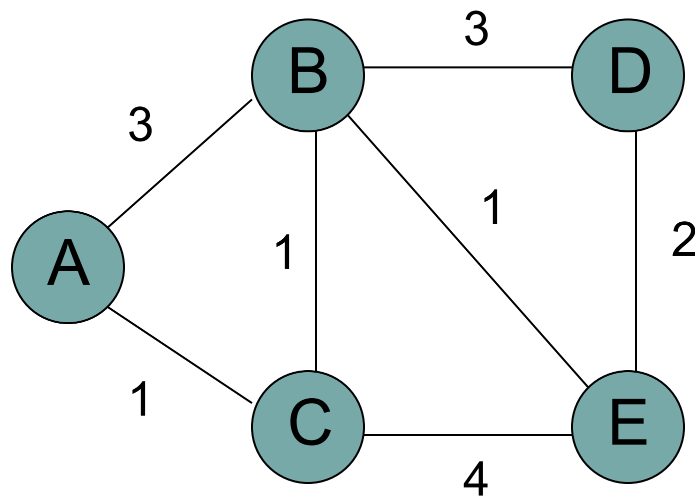

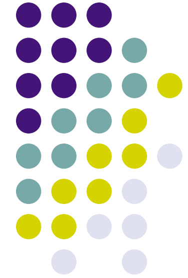

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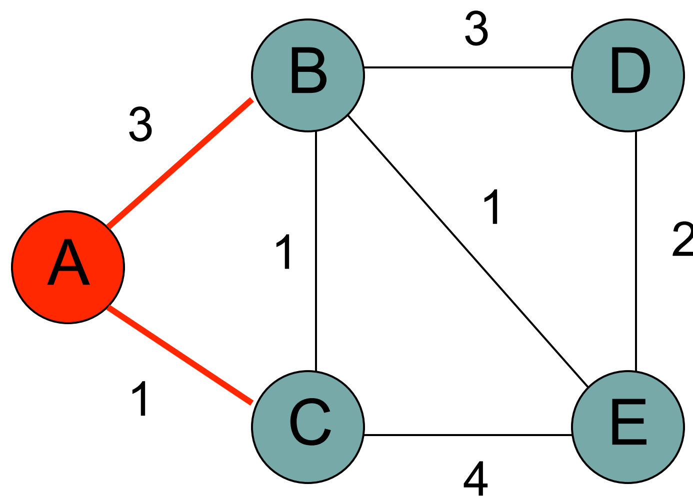

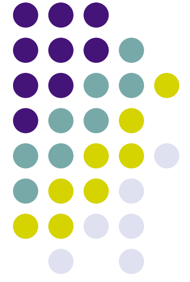

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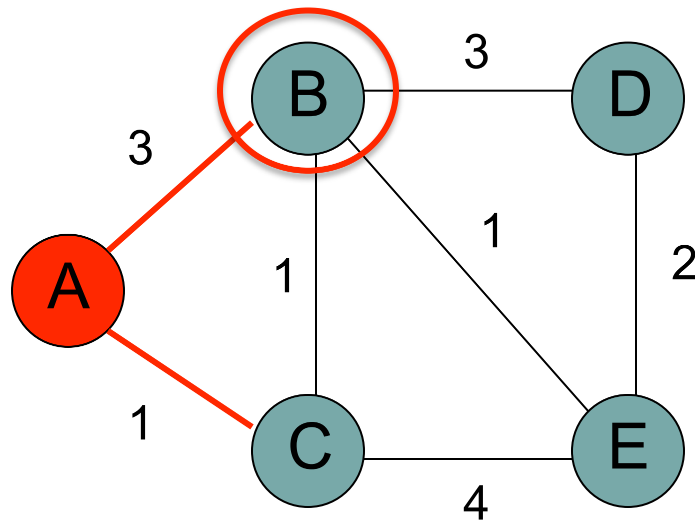


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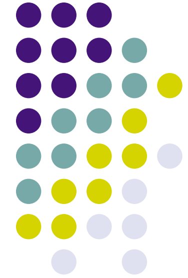

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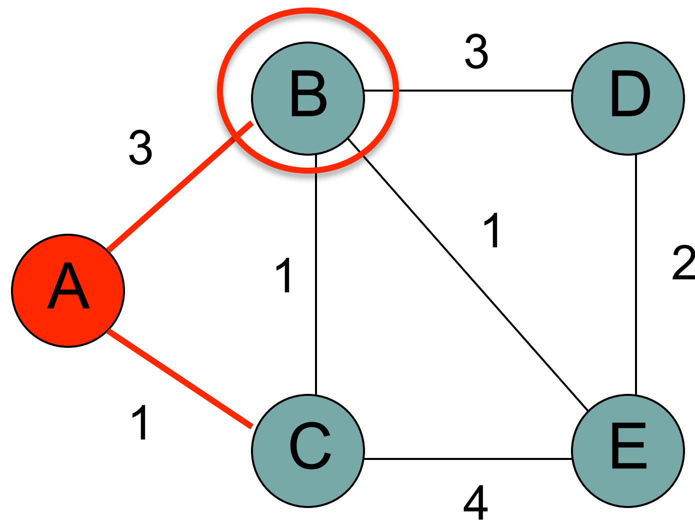


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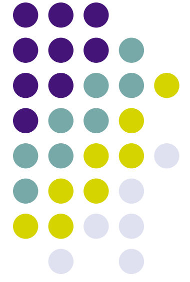
  frontier.push(start, 0);

  while (!frontier.is_empty()) {
    int v = frontier.top_serialnumber();
    int p = frontier.top_priority();
    frontier.pop();

    for (the neighbors (n,w) of v)
      if (n == parents[v])
        ; // do nothing
      else if (n is not in the frontier and has not been visited){
        parents[n] = v;
        frontier.push(n, p + w);
      }else if (p + w < frontier.get_priority(n)) {
        parents[n] = v;
        frontier.reduce_priority(n, p + w);
      }
  } // end while
  return parents;
}
```



Heap	Parent
B 3	A: A
	B: A

```
map<int,int> shortest paths(int start,
                const map<int,list<pair<int,int> > > & graph) {
  map<int,int> parents;
  priorityqueue62 frontier;

  parents[start]=start;
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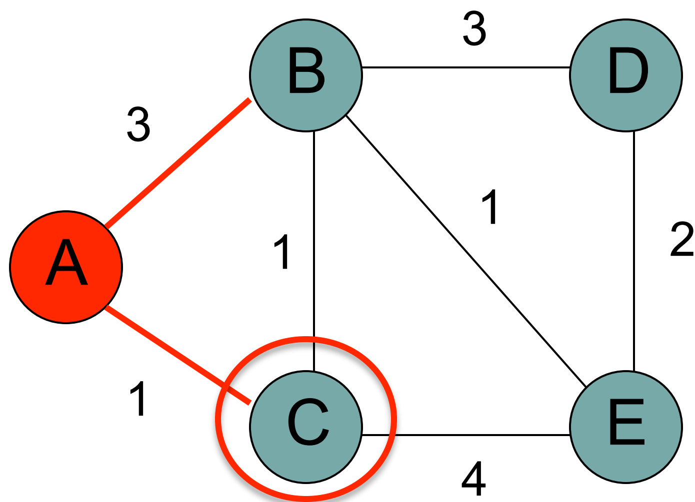

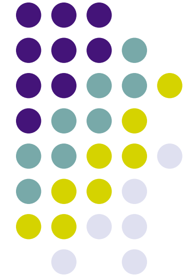

Heap	Parent
C 1	A: A
B 3	B: A
	C: A

```
map<int,int> shortest paths(int start,
                const map<int,list<pair<int,int> > > & graph) {
  map<int,int> parents;
  priorityqueue62 frontier;

  parents[start]=start;
  frontier.push(start, 0);

  while (!frontier.is_empty()) {
    int v = frontier.top_serialnumber();
    int p = frontier.top_priority();
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        frontier.reduce_priority(n, p + w);
      }
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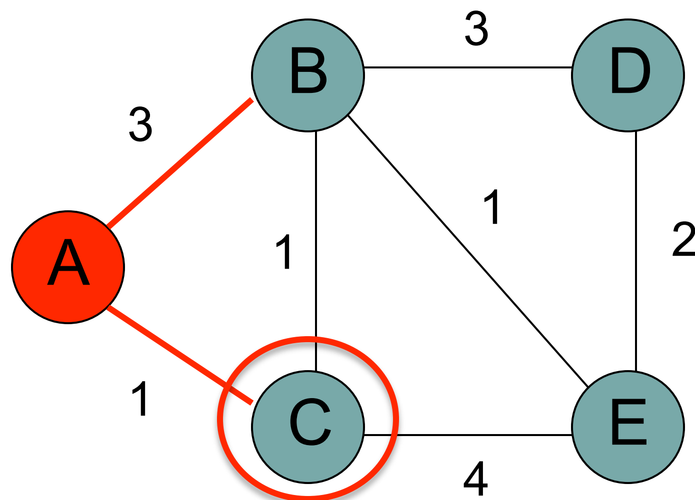
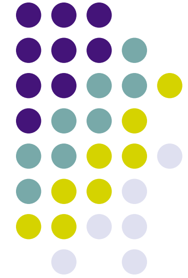


Heap	Parent
B 3	A: A
	B: A
	C: A



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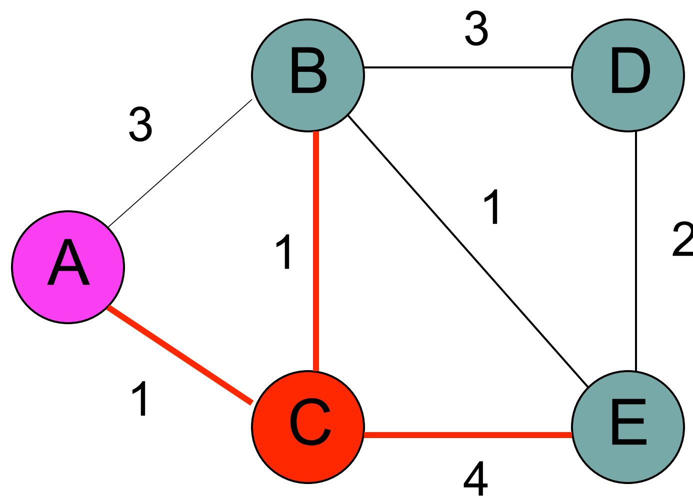

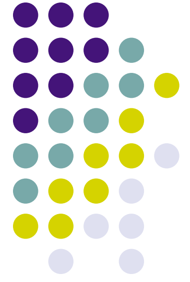

Heap	Parent
B 3	A: A
	B: A
	C: A

```
map<int,int> shortest paths(int start,
                const map<int,list<pair<int,int> > > & graph) {
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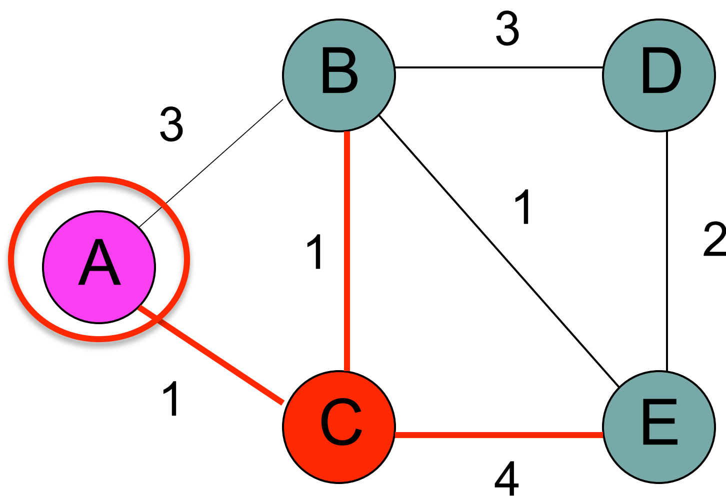
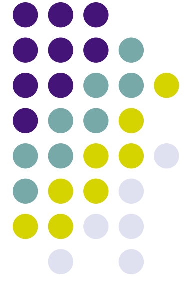
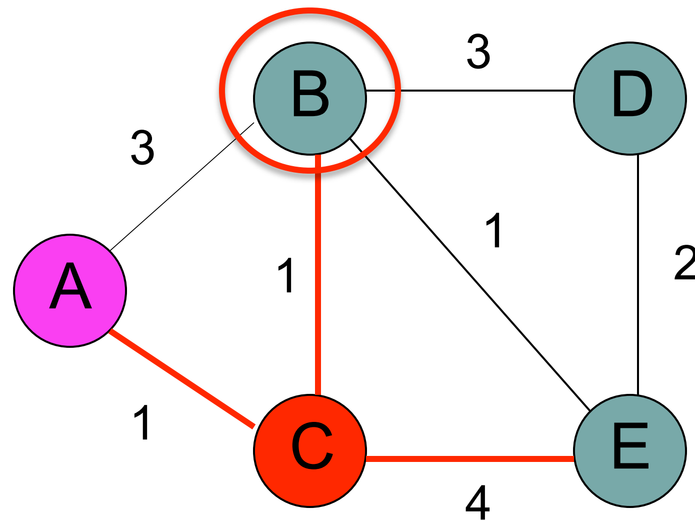


Heap	Parent
B 3	A: A
	B: A
	C: A



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      }
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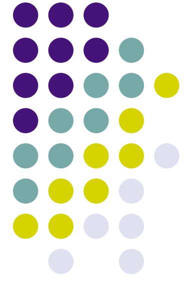
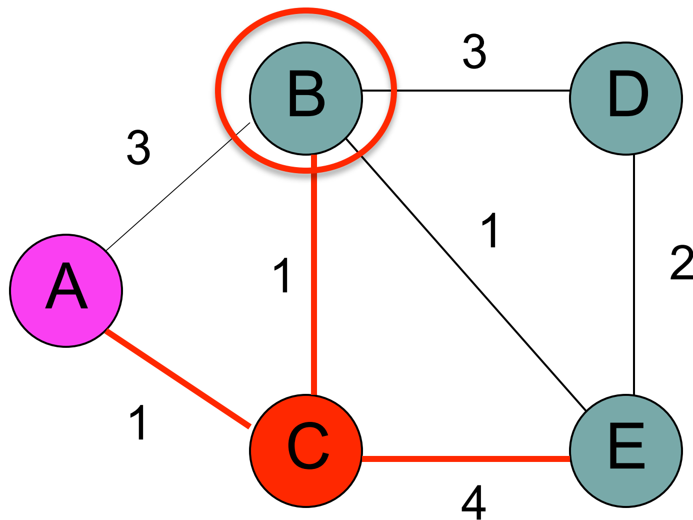


Heap	Parent
B 2	A: A
	B: C
	C: A

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        parents[n] = v;
        frontier.reduce_priority(n, p + w);
      }
    } // end while
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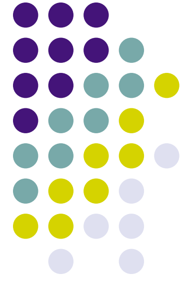
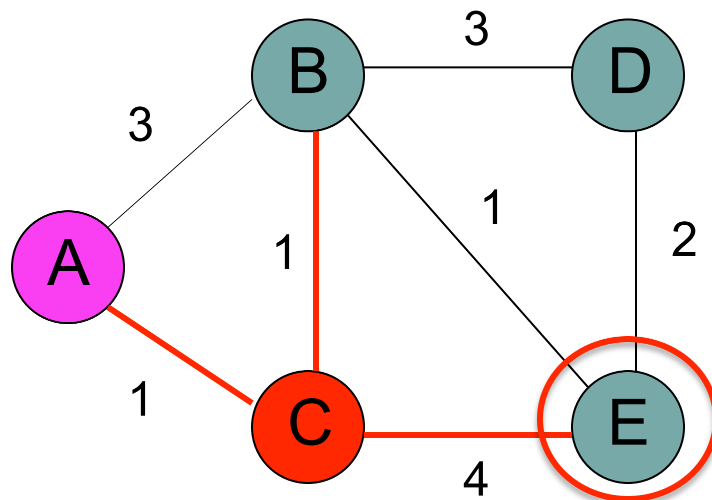


Heap	Parent
B 2	A: A
	B: C
	C: A

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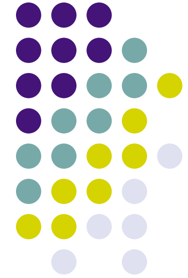
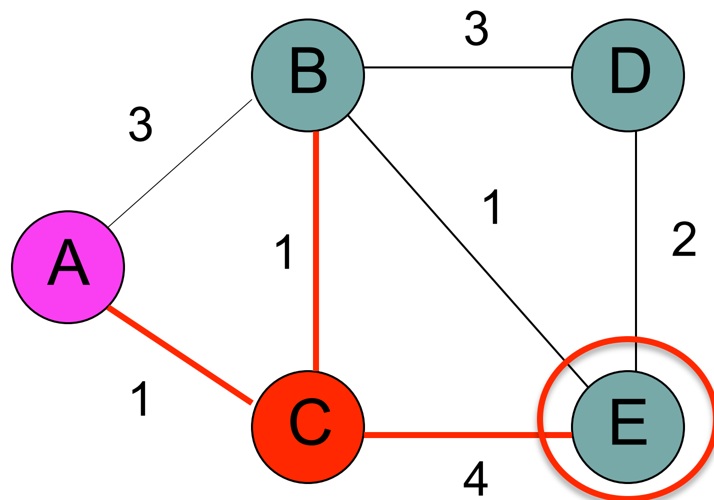


Heap	Parent
B 2	A: A
E 5	B: C
	C: A
	E: C




```
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        parents[n] = v;
        frontier.reduce_priority(n, p + w);
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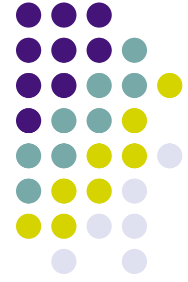
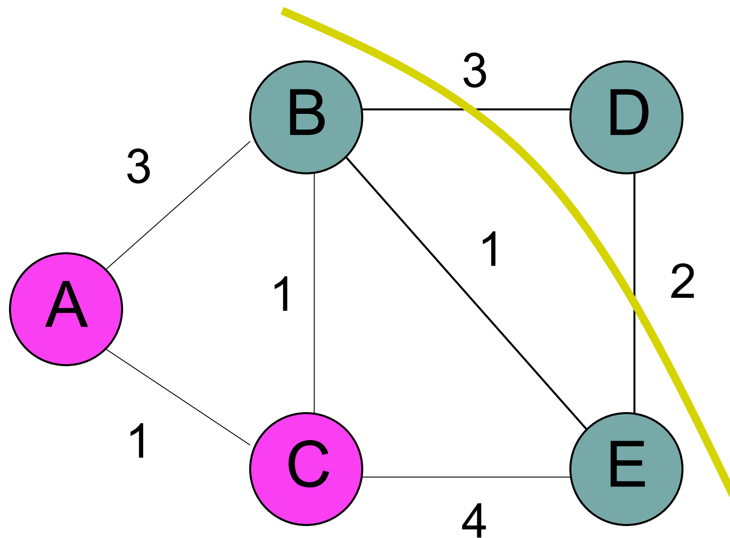


Heap	Parent
B 2	A: A
E 5	B: C
	C: A
	E: C

Frontier: all nodes reachable from starting node within a given distance




```
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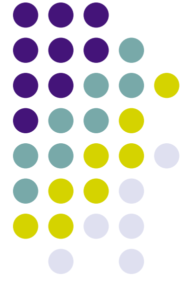
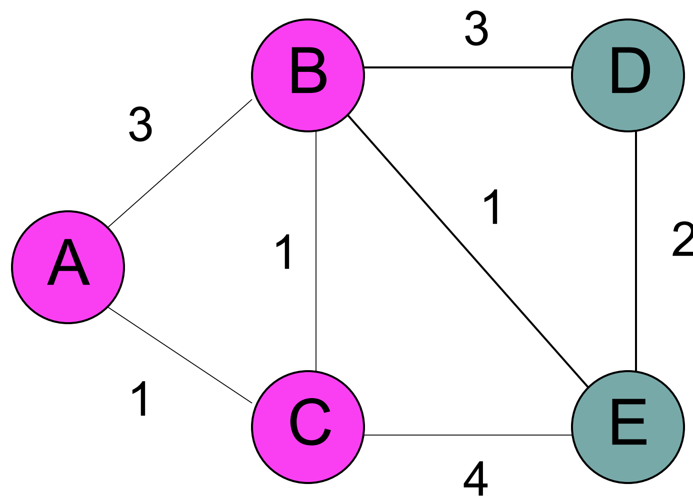


Heap	Parent
E 3	A: A
D 5	B: C
	C: A
	D: B
	E: B




```
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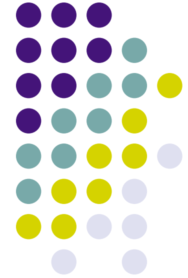
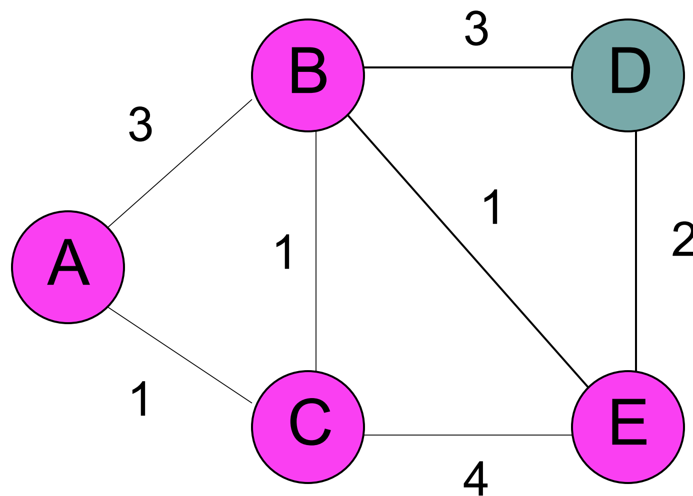


Heap	Parent
D 5	A: A
	B: C
	C: A
	D: B
	E: B

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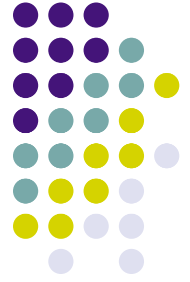

Heap	Parent
	A: A B: C C: A D: B E: B

```
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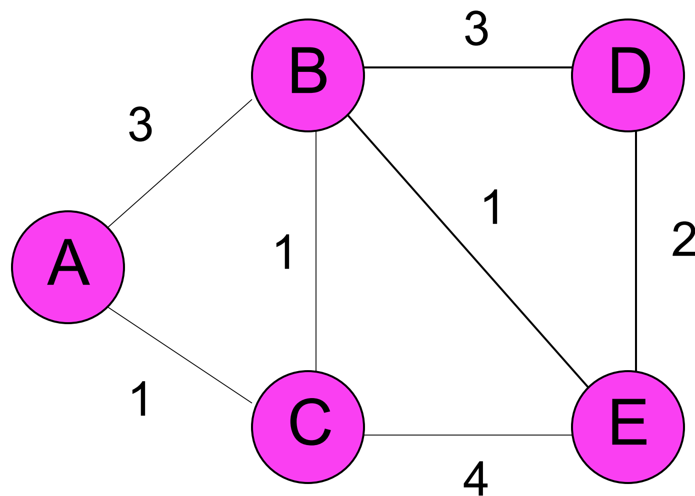

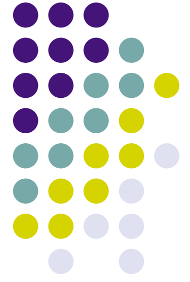

Heap	Parent
	A: A B: C C: A D: B E: B

```
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