CS 151 Set 5 Solutions

1. Exercise 14.14, (a, b, c)

a.

i – NOT true: In particular, I is not independent of B and M.

ii – true: Given G, J is independent of all other nodes, including I.

iii – true: G, B, I is the Markov Blanket for M. Conditioned on its Markov Blanket, a variable is independent of all other variables.

b.

$p(b, i, \neg m, g, j) = p(b)p(\neg m)p(i|b,\neg m)p(g|b,i,\neg m)p(j|g)$

$= 0.9*0.9*0.5*0.8*0.9$

$= 0.2916$

c.

$P(j \mid b, i, m) = \alpha\, P(j, b, i, m)$

$= \alpha \sum_g P(g, j, b, i, m)$

$= \alpha\, p(b)\, p(m)\, p(i|b,m)\sum_g p(g|b,i,m)p(j|g)$

$= \alpha\, p(b)\, p(m)\, p(i|b,m)(p(g|b,i,m)p(j|g) + p(\neg g\, |b,i,m)\, p(j|\neg g))$

$= \text{normalize}(0.9 * 0.1 * 0.9 * (0.9 * 0.9 + 0.1*0),$
$\quad 0.9 * 0.1 * 0.9 * (0.9 * 0.1 + 0.1*1))$

$= \text{normalize}(0.06561, 0.01539)$

$= \langle 0.81, 0.19\rangle$

$= 0.81$

2. Exercise 14.15 (a, b)

$$P(B \mid j.,m)$$

$$= \alpha P(B) \sum_e P(e) \sum_a P(a \mid b,e) P(j \mid a) P(m \mid a)$$

$$= \alpha P(B) \sum_e P(e) \left[.9 \times .7 \times \begin{pmatrix} .95 & .29 \\ .94 & .001 \end{pmatrix} + .05 \times .01 \times \begin{pmatrix} .05 & .71 \\ .06 & .999 \end{pmatrix} \right]$$

$$= \alpha P(B) \sum_e P(e) \begin{pmatrix} .598525 & .183055 \\ .59223 & .0011295 \end{pmatrix}$$

$$= \alpha P(B) \left[.002 \times \begin{pmatrix} .598525 \\ .183055 \end{pmatrix} + .998 \times \begin{pmatrix} .59223 \\ .0011295 \end{pmatrix} \right]$$

$$= \alpha \begin{pmatrix} .001 \\ .999 \end{pmatrix} \times \begin{pmatrix} .59224259 \\ .001493351 \end{pmatrix}$$

$$= \alpha \begin{pmatrix} .00059224259 \\ .0014918576 \end{pmatrix}$$

$$= \langle .284, .716 \rangle$$

Including the normalization, there are 7 additions, 16 multiplications, and 2 divisions. The enumeration algorithm has 2 extra multiplications.