BASIC DATA STRUCTURES

CS062

DATA STRUCTURES AND ADVANCED PROGRAMMING

10: Doubly Linked Lists

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Now that we've (hopefully) understood how singly linked lists work, let's see a very close linear data structure called doubly linked lists.

Lecture 10: Doubly Linked Lists

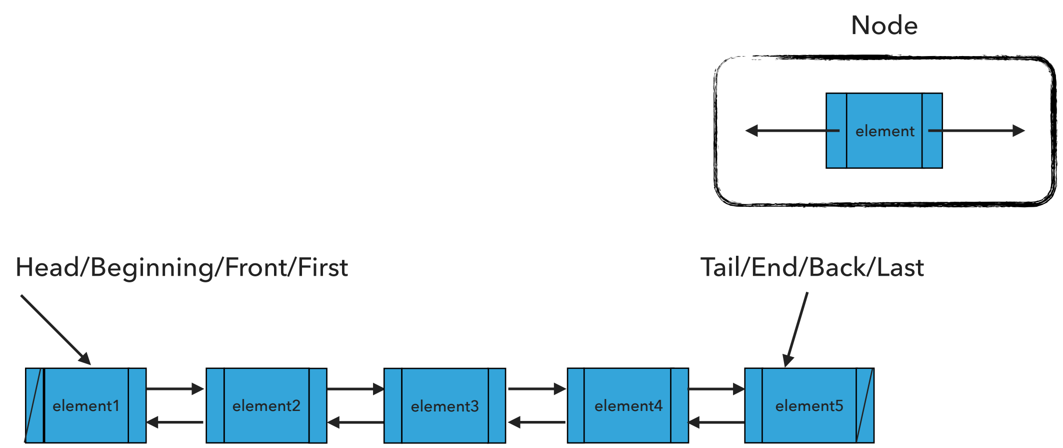
- Doubly Linked Lists
- Java Collections

Some slides adopted from Algorithms 4th Edition and Oracle tutorials

We will start by seeing how we would go about implementing doubly linked lists and we'll finish with the default Java implementation.

Recursive Definition of Doubly Linked Lists

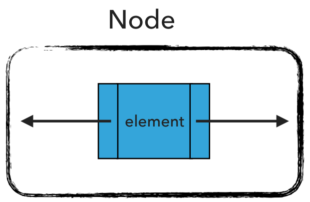
- A doubly linked list is either empty (null) or a node having a reference to a doubly linked list.
- Node: is a data type that holds any kind of data and two references to the previous and next node.



As with singly linked lists, we can recursively define doubly linked lists as either being empty (null) or a node having a reference to a doubly linked list. A node is a data type that holds any kind of data (think generics) but instead of one reference it now has two, one to the previous and one to the next node. We will use arrows to symbolize the links, rectangles for the nodes, and we will use slashes to indicate the first node called head/beginning/front/or first and the last node, called tail/end/back and last.

Node

```
private class Node {
    E element;
    Node next;
    Node prev;
  }
```



Nodes will be again represented through a private inner class that contains an element of type E and two reference to the previous and next node.

Reminder: Interface List

```
public interface List <E> {
    void add(E element);
    void add(int index, E element);
    void clear();
    E get(int index);
    boolean isEmpty();
    E remove();
    E remove(int index);
    E set(int index, E element);
    int size();
}
```

Let’s refresh our memory about the List interface. If we implement it, we promise to implement the methods:

void add(E element);
void add(int index, E element);
void clear();
E get(int index);
boolean isEmpty();
E remove();
E remove(int index);
E set(int index, E element);

Standard Operations

- `DoublyLinkedList()`: Constructs an empty doubly linked list.
- `isEmpty()`:Returns true if the doubly linked list does not contain any element.
- `size()`: Returns the number of elements in the doubly linked list.
- E `get(int index)`: Returns the element at the specified index.
- `addFirst(E element)`: Inserts the specified element at the head of the doubly linked list.
- `addLast(E element)`: Inserts the specified element at the tail of the doubly linked list.
- `add(E element)`: Inserts the specified element at the tail of the doubly linked list.
- `add(int index, E element)`: Inserts the specified element at the specified index.
- E `set(int index, E element)`: Replaces the specified element at the specified index and returns the old element
- `E removeFirst()`: Removes and returns the head of the doubly linked list.
- `E removeLast()`: Removes and returns the tail of the doubly linked list.
- `E remove()`: Removes and returns the head of the doubly linked list.
- `E remove(int index)`: Removes and returns the element at the specified index.
- `clear()`: Removes all elements.

These are the standard operations we expect to have. We will have a constructor and usual methods for checking the size, whether it is empty, a getter, three adds, one set, three removes, and one clear.

`DoublyLinkedList()`: Constructs an empty DLL

head

tail

size

What should happen?

```
DoublyLinkedList<String> dll = new DoublyLinkedList<String>();
```

Let's say someone creates a doubly linked lists of strings. what do you think should happen to the head, tail, and size?

`DoublyLinkedList()`: Constructs an empty DLL

```
DoublyLinkedList<String> dll = new DoublyLinkedList<String>();
```

head = null

tail = null

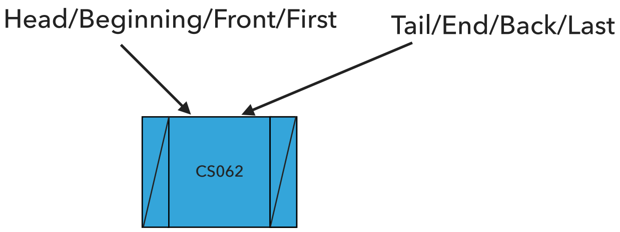
size = 0

What should happen?

`dll.add("CS062");`

the head and tail will be null and the size zero. What would happen if we call dll.add(“CS062”);

`add(E element)`: Inserts the specified element at the tail of the doubly linked list.



`dll.add("CS062")`

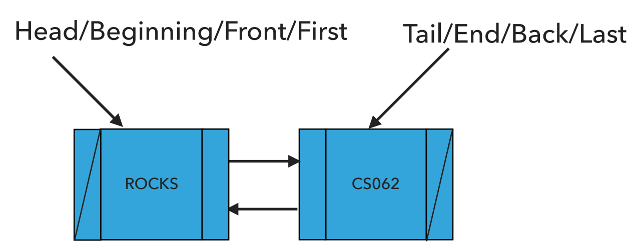
`size=1`

What should happen?

`dll.addFirst("ROCKS");`

The head and tail point to the node that contains CS062 and the size is 1. What if we call dll.addFirst("ROCKS");

`addFirst(E element)`: Inserts the specified element at the head of the doubly linked list



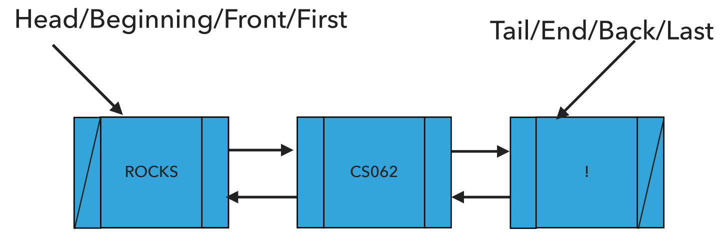

```
dll.addFirst("ROCKS")

size=2
```

What should happen?

```
dll.addLast("!");
```

The addition will happen at the head. The head now points to the node that contains ROCKS while the tail points to cs062. The size is 2. What should happen if we type dll.addLast("!");

`addLast(E element)`: Inserts the specified element at the tail of the doubly linked list



`dll.addLast("!")`

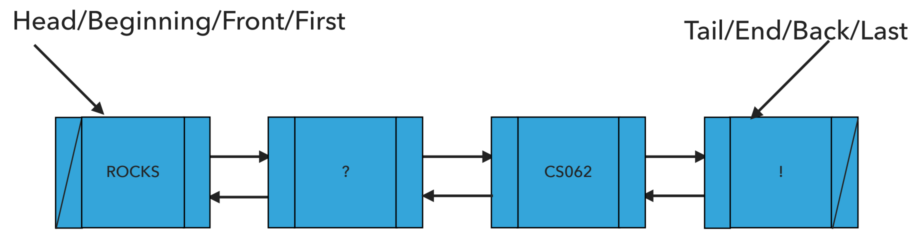
`size=3`

What should happen?

`dll.add(1,"?");`

The addition happens at the end. The head remains pointing to the node that contains ROCKS. The insertion creates a new node that contains ! and the tail is moved to it. The size is now 3. What should happen if we call dll.add(1,"?");

`add(int index, E element)`: Adds element at the specified index



`dll.add(1,"?")`

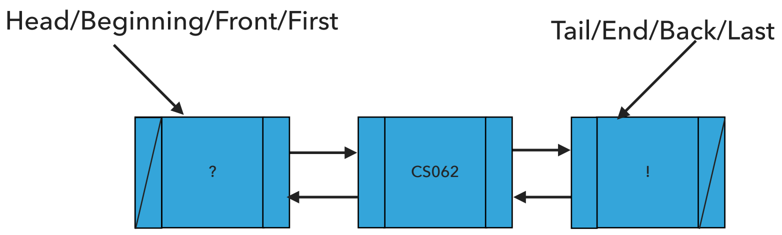
`size=4`

What should happen?

`dll.remove();`

We will make room for a new node to be created at index 1 and increase the size by 1. What if we call remove?

`remove()`: Removes and returns the head of the doubly linked list



`dll.remove()`

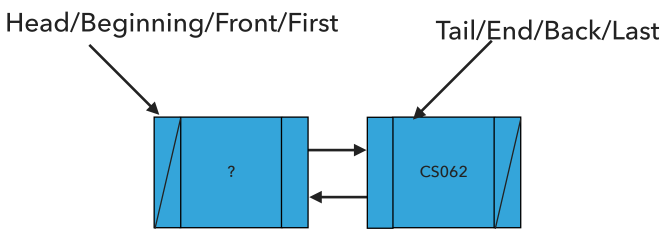
`size=3`

What should happen?

`dll.removeLast();`

remove removes and returns the old head of the doubly linked list while moving the head pointer to the next node. And of course reduces the size by 1. removeFirst works exactly the same way.How about removeLast?

`removeLast()`: Removes and returns the tail of the doubly linked list



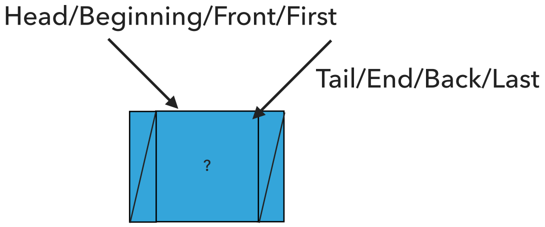

```
dll.removeLast()

size=2
```

What should happen?

```
dll.remove(1);
```

The removal happens at the tail and the size reduces by 1. How about remove(1)?

`remove(int index)`: Removes and returns the element at the specified index




```
dll.remove(1)

size=1
```

The node at index 1 (second node) will be removed and the size will be reduced by 1.

Our own implementation of Doubly Linked Lists

- We will follow the recommended textbook style.
  - It does not offer a class for this so we will build our own.
- We will work with generics because we want doubly linked lists to hold objects of an type.
- We will implement the List interface we defined in past lectures.
- We will use an inner class Node and we will keep track of how many elements we have in our doubly linked list.

Our own implementation of doubly linked lists will lead us to work with generics. we will use the list interface and an inner class for nodes.

Instance variables and inner class

```
public class DoublyLinkedList<E> implements List<E> {
   private Node head; // head of the doubly linked list
   private Node tail; // tail of the doubly linked list
   private int size; // number of nodes in the doubly linked list

   /**
    * This nested class defines the nodes in the doubly linked list with a value
    * and pointers to the previous and next node they are connected.
    */
   private class Node {
      E element;
      Node next;
      Node prev;
   }
```

That means that we will have three instance variables, head and tail of type Node, and size of type int along with our inner private class for Node.

Check if is empty and how many elements

```
/**
 * Returns true if the doubly linked list does not contain any element.
 *
 * @return true if the doubly linked list does not contain any element
 */
public boolean isEmpty() {
    return size == 0; // or return (head == null && tail == null);
}

/**
 * Returns the number of elements in the doubly linked list.
 *
 * @return the number of elements in the doubly linked list
 */
public int size() {
    return size;
}
```

isEmpty can either check whether the head and tail is null or the size 0. size is very simple.

PRACTICE TIME: Retrieve element from specified index

```
/**
 * Returns element at the specified index.
 *
 * @param index
 *            the index of the element to be returned
 * @return the element at specified index
 */
public E get(int index) {
    // check whether index is valid

    // if index is 0, return element at head


     // else if index is size-1, return element at tail


    // set a temporary pointer to the head

    // search for index-th element or end of list


    // return the element stored in the node that the temporary pointer points to
}
```

Knowing what we know about pointers, let's try to implement get.

Retrieve element from specified index

```
/**
 * Returns element at the specified index.
 *
 * @param index
 *            the index of the element to be returned
 * @return the element at specified index
 * @pre 0<=index<size
 */
public E get(int index) {
    // check whether index is valid
     if (index >= size || index < 0){
          throw new IndexOutOfBoundsException("Index " + index + " out of bounds");
     }
     // if index is 0, return element at head
    if (index == 0){
        return head.element;
     }
     // else if index is size-1, return element at tail
    else if (index == size - 1){
        return tail.element;
     }
    // set a temporary pointer to the head
    Node finger = head;
    // search for index-th element or end of list
    while (index > 0) {
        finger = finger.next;
        index--;
    }
    // return the element stored in the node that the temporary pointer points to
    return finger.element;
}
```

The get method will check that the index is within bounds and if not will throw an exception. We will next use the usual trick: we will create a reference that points to where head points to (NOT A NEW NODE!) We will move index steps to the right by pointing finger to finger.next. Eventually, when finger points to the right node, we will return the element it holds.

PRACTICE TIME: Insert element at head of doubly linked list

```
/**
 * Inserts the specified element at the head of the doubly linked list.
 *
 * @param element
 *            the element to be inserted
 */
public void addFirst(E element) {
    // Create a pointer to head

    // Make a new node and assign it to head. Fix pointers and update element




    // if first node to be added, adjust tail to it.


    // else fix previous pointer to head

    // increase number of nodes in doubly linked list.

}
```

Let's try addFirst to add an element to the head of the doubly linked list.

Insert element at head of doubly linked list

```java
/**
 * Inserts the specified element at the head of the doubly linked list.
 *
 * @param element
 *            the element to be inserted
 */
public void addFirst(E element) {
    // Create a pointer to head
    Node oldHead = head;

    // Make a new node and assign it to head. Fix pointers and update element
    head = new Node();
    head.element = element;
    head.next = oldHead;
    head.prev = null;

    // if first node to be added, adjust tail to it.
    if (tail == null){
        tail = head;
     }
    else{
        // else fix previous pointer to head
        oldHead.prev = head;
     }
    // increase number of nodes in doubly linked list.
     size++;
 }
```

Did you get something similar?

PRACTICE TIME: Insert element at tail of doubly linked list

```java
/**
 * Inserts the specified element at the tail of the doubly linked list.
 *
 * @param element
 *            the element to be inserted
 */
public void addLast(E element) {
    // Create a pointer to tail

    // Make a new node and assign it to tail. Fix pointers and update element



    // if first node to be added, adjust head to it.


    // else fix next pointer to tail

    // increase number of nodes in doubly linked list.

}
```

How about adding to the tail?

Insert element at tail of doubly linked list

```
/**
 * Inserts the specified element at the tail of the doubly linked list.
 *
 * @param element
 *            the element to be inserted
 */
public void addLast(E element) {
    // Create a pointer to tail
    Node oldTail = tail;

    // Make a new node and assign it to tail. Fix pointers and update element
    tail = new Node();
    tail.element = element;
    tail.next = null;
    tail.prev = oldTail;

    // if first node to be added, adjust head to it.
    if (head == null)
        head = tail;
    else{
         // else fix next pointer to tail
        oldTail.next = tail;
     }
    // increase number of nodes in doubly linked list.
    size++;
}
```

It should look familiar.

Insert element at tail of doubly linked list

```java
/**
 * Inserts the specified element at the tail of the doubly linked list.
 *
 * @param element
 *            the element to be inserted
 */
public void add(E element) {
    // Create a pointer to tail
    addLast(element);

}
```

As a note, add is just a call to addLast.

PRACTICE TIME: Insert element at a specified index

```
/**
 * Inserts the specified element at the specified index.
 *
 * @param index
 *            the index to insert the element
 * @param element
 *            the element to insert
 * @pre 0<=index<=size
 */
public void add(int index, E element) {
        // check whether index is valid

         // if index is 0, call addFirst


         // if index is size, call addLast


         // else
              // Make two new Node references, previous and finger. Set previous to null and finger to head


          // search for index-th position. Set previous to finger and move finger to next position



          // create new Node, update its element, and fix its pointers taking into account where finger and previous
are


                // increase number of nodes

}
```

Let's try to add at a specific index.

Insert element at a specified index

```java
/**
 * Inserts the specified element at the specified index.
 *
 * @param index
 *            the index to insert the element
 * @param element
 *            the element to insert
 * @pre 0<=index<=size
 */
public void add(int index, E element) {
       // check whether index is valid
       if (index > size || index < 0){
           throw new IndexOutOfBoundsException("Index " + index + " out of bounds");
        }
        // if index is 0, call addFirst
     if (index == 0) {
          addFirst(element);
        // if index is n, call addLast
     } else if (index == size()) {
          addLast(element);
        // else
     } else {
              // Make two new Node references, previous and finger. Set previous to null and finger to head
          Node previous = null;
          Node finger = head;
          // search for index-th position. Set previous to finger and move finger to next position
          while (index > 0) {
               previous = finger;
               finger = finger.next;
               index--;
          }
          // create new Node, update its element, and fix its pointers taking into account where finger and previous are
          Node current = new Node();
          current.element = element;
          current.next = finger;
          current.prev = previous;
          previous.next = current;
          finger.prev = current;
              // increase number of nodes
          size++;
     }
```

This is more work. We will need to double down on our trick and have two pointers. Let's call them previous and finger. Finger will start at the head and previous will trail it one step behind. Eventually, finger will reach the index we want to insert the new node which we will reference with current. We will use these two pointers, to make the previous point to current (and vice versa), and current point to finger (and vice versa).

Replace element at a specified index

```
/**
 * Inserts the specified element at the specified index.
 *
 * @param index
 *            the index of the element to replace
 * @param element
 *            the element to be stored at the specific index
 * @return the old element that was replaced
 * @pre 0<=index<size
 */
public E set(int index, E element) {
        // check that index is within range
      if (index >= size || index < 0){
            throw new IndexOutOfBoundsException("Index " + index + " out of bounds");
        }

        Node finger = head;
        // search for index-th position by pointing previous to finger and advancing finger
        while (index > 0) {
              finger = finger.next;
              index--;
         }
         // reference old element
         E old = finger.element;
         // update element at finger
         finger.element = element;
         // return old element
         return old;
    }
}
```

Replacing an element a specified index will look exactly the same as with singly linked lists.

PRACTICE: Retrieve and remove head

```
/**
 * Removes and returns the head of the doubly linked list.
 *
 * @return the head of the doubly linked list.
 */
public E removeFirst() {
    // Create a pointer to head

    // Move head to next

    // if there was only one node left in doubly linked list

        // remove tail by setting it to null


    // else

        // set previous pointer of head to null

    // set old head's next pointer to null

    // decrease number of nodes

    // return old head's element

}
```

Let's try to remove the head.

Retrieve and remove head

```
/**
 * Removes and returns the head of the doubly linked list.
 *
 * @return the head of the doubly linked list.
 */
public E removeFirst() {
    // Create a pointer to head
    Node oldHead = head;
    // Move head to next
    head = head.next;
      // if there was only one node in the doubly linked list.
    if (head == null) {
        tail = null
    } else {
      head.prev= null;
    }
    // decrease number of nodes
    size--;
    // return old head's element
    return oldHead.element;
}
```

Did you get something like this? Don't forget that removing the last node is an edge case we need to handle by fixing the tail.

PRACTICE TIME: Retrieve and remove tail

```
/**
 * Removes and returns the tail of the doubly linked list.
 *
 * @return the tail of the doubly linked list.
 */
public E removeLast() {
    // Create a pointer to tail

    // Move tail to previous

    // if removed the last node

        // set head to null

    // else

        // set new tail’s next to null
    }
    // decrease number of nodes

    // return old tail’s element

}
```

How about removeLast?

Retrieve and remove tail

```
/**
 * Removes and returns the tail of the doubly linked list.
 *
 * @return the tail of the doubly linked list.
 */
public E removeLast() {
    // Create a pointer to tail
    Node temp = tail;
    // Move tail to previous
    tail = tail.prev;
    // if removed the last node
    if (tail == null) {
        // set head to null
        head = null;
    // else
    } else {
        // set new tail’s next to null
        tail.next = null;
    }
    // decrease number of nodes
    size--;
    // return old tail’s element
    return temp.element;
}
```

Very similar idea.

Retrieve and remove head

```
/**
 * Removes and returns the head of the doubly linked list.
 *
 * @return the head of the doubly linked list.
 */
public E remove() {
    return removeFirst();
}
```

remove is just a call to removeFirst.

PRACTICE TIME: Retrieve and remove element from a specific index

```
/**
 * Removes and returns the element at the specified index.
 *
 * @param index
 *            the index of the element to be removed
 * @return the element previously at the specified index
 * @pre 0<=index<size
 */
public E remove(int index) {
       // check whether index is valid

      // if index is 0

             // return removeFirst

      // else if index is size-1

             // return removeLast

      // else
             // Make two new Node references, previous and finger. Set previous to null and finger to head

           // search for index-th position. Set previous to finger and move finger to next position


           // update pointers for previous and finger


           // decrease number of nodes

           // return the element that finger points to

      }

}
```

What about removing at a specific index?

Retrieve and remove element from a specific index

```
/**
 * Removes and returns the element at the specified index.
 *
 * @param index
 *            the index of the element to be removed
 * @return the element previously at the specified index
 * @pre 0<=index<size
 */
public E remove(int index) {
      // check whether index is valid
      if (index >= size || index < 0){
          throw new IndexOutOfBoundsException("Index " + index + " out of bounds");
      }
      // if index is 0
     if (index == 0) {
            // return removeFirst
          return removeFirst();
      // else if index is size-1
     } else if (index == size - 1) {
            // return removeLast
          return removeLast();
      // else
     } else {
            // Make two new Node references, previous and finger. Set previous to null and finger to head
          Node previous = null;
          Node finger = head;
          // search for index-th position. Set previous to finger and move finger to next position
          while (index > 0) {
               previous = finger;
               finger = finger.next;
               index--;
          }
          // update pointers for previous and finger
          previous.next = finger.next;
          finger.next.prev = previous;
          // decrease number of nodes
          size--;
          // return the element that finger points to
          return finger.element;
     }

}
```

We will use again the same trick to find the node at the index we want to remove it.

Clear the singly linked list of all elements

```
/**
 * Clears the doubly linked list of all elements.
 *
 */
public void clear(
        head = null;
        tail = null;
        size = 0;

}
```

Clear is super simple. Just set the head and tail to null and the size to 0. The garbage collector will take care of the rest.

`addFirst()` in doubly linked lists is $O(1)$ for worst case

```
public void addFirst(E element) {
      // Save the old node
      Node oldHead = head;

      // Make a new node and assign it to head. Fix pointers.
      head = new Node();
      head.element = element;
      head.next = oldHead;
      head.prev = null;

      // if first node to be added, adjust tail to it.
      if (tail == null)
         tail = head;
      else
         oldHead.prev = head;

      size++; // increase number of nodes in doubly linked list.
   }
```

Let's look now into the running time complexity of addFirst. It will be O(1). It does not depend on how many elements already exist in the doubly linked list. The fact that we need to do a couple of operations doesn't matter. they don't scale linearly with the size of the doubly linked list.

`addLast()` in doubly linked lists is $O(1)$ for worst case

```
public void addLast(E element) {
      // Save the old node
      Node oldTail = tail;

      // Make a new node and assign it to tail. Fix pointers.
      tail = new Node();
      tail.element = element;
      tail.next = null;
      tail.prev = oldTail;

      // if first node to be added, adjust head to it.
      if (head == null)
         head = tail;
      else
         oldTail.next = tail;

      size++;
   }
```

Same idea for addLast (and as a consequence for add)

get(int index) in doubly linked lists is $O(n)$ for worst case

```
/**
 * Returns element at the specified index.
 *
 * @param index
 *            the index of the element to be returned
 * @return the element at specified index
 * @pre 0<=index<size
 */
public E get(int index) {
    // check whether index is valid
     if (index >= size || index < 0){
          throw new IndexOutOfBoundsException("Index " + index + " out of bounds");
     }
     // if index is 0, return element at head
    if (index == 0){
        return head.element;
     }
     // else if index is size-1, return element at tail
    else if (index == size - 1){
        return tail.element;
     }
    // set a temporary pointer to the head
    Node finger = head;
    // search for index-th element or end of list
    while (index > 0) {
        finger = finger.next;
        index--;
    }
    // return the element stored in the node that the temporary pointer points to
    return finger.element;
}
```

Get is another story. It can take O(n) for worst case if we need to hop n steps to find the desired index.

`add(int index, E element)` in doubly linked lists is $O(n)$ for worst case

```
public void add(int index, E element) {
        if (index == 0) {
            addFirst(element);
        } else if (index == size()) {
            addLast(element);
        } else {

            Node previous = null;
            Node finger = head;
            // search for index-th position
            while (index > 0) {
                previous = finger;
                finger = finger.next;
                index--;
            }
            // create new value to insert in correct position
            Node current = new Node();
            current.element = element;
            current.next = finger;
            current.prev = previous;
            previous.next = current;
            finger.prev = current;

            size++;
        }
    }
```

same idea for add, worst case is O(n).

`set(int index, E element)` in singly linked lists is $O(n)$ for worst case

```
/**
 * Inserts the specified element at the specified index.
 *
 * @param index
 *            the index of the element to replace
 * @param element
 *            the element to be stored at the specific index
 * @return the old element that was replaced
 * @pre 0<=index<size
 */
public E set(int index, E element) {
        // check that index is within range
      if (index >= size || index < 0){
            throw new IndexOutOfBoundsException("Index " + index + " out of bounds");
        }

        Node finger = head;
        // search for index-th position by pointing previous to finger and advancing finger
        while (index > 0) {
              finger = finger.next;
              index--;
         }
         // reference old element
         E old = finger.element;
         // update element at finger
         finger.element = element;
         // return old element
         return old;
    }
}
```

and for set.

`removeFirst()` in doubly linked lists is $O(1)$ for worst case

```
public E removeFirst() {
      Node oldHead = head;
      // Fix pointers.
      head = head.next;
      // if there was only one node in the doubly linked list.
      if (head == null) {
         tail = null
      } else {
          head.next= null;
      }

      size--;

      return oldHead.element;
   }
```

removeFirst (and remove) from the head in contrast is O(1) like with addFirst.

`removeLast()` in doubly linked lists is $O(1)$ for worst case

```
public E removeLast() {

    Node temp = tail;
    tail = tail.prev;

    // if there was only one node in the doubly linked list.
    if (tail == null) {
        head = null;
    } else {
        tail.next = null;
    }
    size--;
    return temp.element;
}
```

Same idea for removeLast

`remove(int index)` in doubly linked lists is $O(n)$ for worst case

```
public E remove(int index) {
    if (index == 0) {
        return removeFirst();
    } else if (index == size() - 1) {
        return removeLast();
    } else {
        Node previous = null;
        Node finger = head;
        // search for value indexed, keep track of previous
        while (index > 0) {
            previous = finger;
            finger = finger.next;
            index--;
        }
        previous.next = finger.next;
        finger.next.prev = previous;

        size–;
        // finger's value is old value, return it
        return finger.element;
    }

}
```

But remove at a specific index can be O(n)

Lecture 10: Doubly Linked Lists

- Doubly Linked Lists
- Java Collections

`clear()` in singly linked lists is $O(1)$ for worst case

```
/**
 * Clears the doubly linked list of all elements.
 *
 */
public void clear(
        head = null;
        tail = null;
        size = 0;

}
```

Clear is O(1)!

Lecture 10: Doubly Linked Lists

- Doubly Linked Lists
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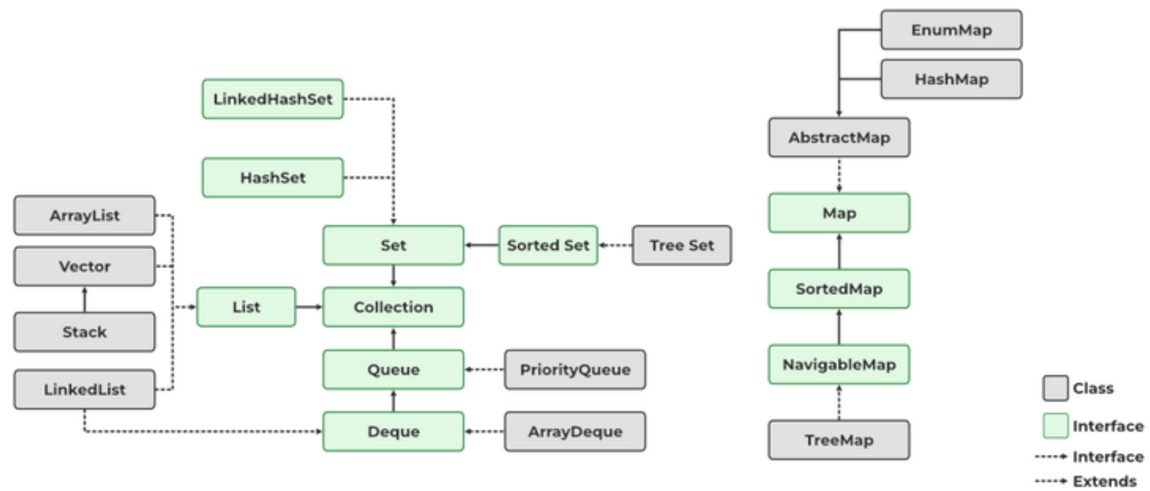
That's all for our own implementation. Let's see Java's default implementation

The Java Collections Framework



https://www.geeksforgeeks.org/collections-in-java-2/

LinkedList also implements the list interface like array list and vector.

LinkedList in Java Collections

- Doubly linked list implementation of the List and Deque (stay tuned) interfaces.

```
java.util.LinkedList;

public class LinkedList<E> extends
AbstractSequentialList<E> implements List<E>, Deque<E>
```

https://docs.oracle.com/javase/7/docs/api/java/util/LinkedList.html

If you want to use it, you will have to import the java.util.LinkedList;

Lecture 10: Doubly Linked Lists

- Doubly Linked Lists
- Java Collections

And that's all for today

Readings:

- Oracle's guides:
  - Collections: https://docs.oracle.com/javase/tutorial/collections/intro/index.html
  - Linked Lists: https://docs.oracle.com/javase/7/docs/api/java/util/LinkedList.html
- Recommended Textbook:
  - Chapter 1.3 (Page 142-146)
- Recommended Textbook Website:
  - Linked Lists: https://algs4.cs.princeton.edu/13stacks/

Code

- Lecture 10 code

Practice Problems:

- 1.3.18–1.3.27 (approach them as doubly linked lists).

Feel free to download the code and play with our implementation.