In-Class Worksheet

Discrete Math & Functional Programming— CSCI 054— Spring 2025

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Let $T(n)$ be the number of filled triangles in a Sierpinski's triangle after n interations where $T(0)$ is a single filled triangle. Observe that $T(n) = 3T(n-1)$

Use induction to prove that $T(n) = 3^n$.

Consider the recurrence relation:

$$\begin{aligned}T(n) &= 5T(n-1) - 6T(n-2)\\ T(0) &= 2\\ T(1) &= 5\end{aligned}$$

Claim: $\forall n \in \mathbb{Z}_0^+ : T(n) = 2^n + 3^n$

- We prove the claim using a proof by strong induction on:
- Base case(s):
- Inductive hypothesis (IHOP):
- Inductive step:
  - We want to show that:
  - Proof:
- Therefore by the principle of mathematical induction: