csci54 - discrete math \& functional programming propositional logic continued, predicate logic

## last time

- introduction to propositional logic:
- Boole
- proposition
- well-formed propositional logic formulas (wff)

$$
\phi::=T|F|(\neg \phi)|(\phi \wedge \phi)|(\phi \vee \phi) \mid(\phi \Rightarrow \phi)
$$

- truth tables for operators
- tautology/satisfiable/contingency (falsifiable)/contradiction
- implication
- logical equivalence


## converse, inverse, contrapositive

Given an implication $p \Rightarrow q$, we can derive three other implications:

E converse: $q \rightarrow p$
E inverse: $\neg \mathrm{p} \rightarrow \neg \mathrm{q}$
contrapositive: $\neg q \rightarrow \neg p$

- Which, if any, of the converse, inverse, and contrapositive is logically equivalent to the original implication?


## consider the following statements . . .

- If 2 is an even number then 3 is an odd number.
- If $x$ is an even number, then $x+1$ is an odd number.
- How would you express these two statements in propositional logic?


## predicate logic

- A predicate $P$ is function that assigns the value True or False to each element of a set $U$.
- The set $U$ is called the universe or domain of discourse
- $P$ is a predicate over $U$
- Examples:
" the predicate "is an even number" over the positive integers.
- the predicate "last name has at least 6 characters" over the set of people currently in this room.
- Once you specify the element of $U$, then you have a proposition with a truth value.


## quantifiers

- quantifiers are another way to form propositions from a predicate
Definition 3.21: Universal quantifier [for all, $\forall$ ].
Let $P$ be a predicate over $S$. The proposition $\forall x \in S: P(x)$ is true if, for every possible $x \in S$, we have that $P(x)$ is true.

Definition 3.22: Existential quantifier [there exists, $\exists$ ].
Let $P$ be a predicate over $S$. The proposition $\exists x \in S: P(x)$ is true if, for at least one possible $x \in S$, we have that $P(x)$ is true.

## quantifiers - example

- Imagine these predicates
- "rested(n)" means "n got at least 8 hours of sleep in the past 24 hours"
- "bornMA(n)" means "n was born in Massachusetts"
- Which, if any, of the following propositions is true? Justify your answer.
- $\forall \mathrm{n}$ in this room : rested(n)
- $\forall \mathrm{n}$ in this room : (rested(n) $\wedge$ bornMA(n))
- $\exists \mathrm{n}$ currently enrolled at Pomona College : (rested(n) v bornMA(n))
- ヨ n currently enrolled at Pomona College : (rested(n) ^ bornMA(n))


## free and bound variables (an aside)

- In an expression variables can be free/unbound or bound
- With a free variable the value is not fixed by the expression
- With a bound variable the value is defined within the expression

$$
\forall x \in \mathbb{Z}: x^{2} \geq y
$$

- An expression of predicate logic with no free variables is called fully quantified


## theorems in predicate logic

- A fully quantified expression of predicate logic is a theorem if and only if it is true for every possible meaning of each of its predicates.
- Is the following a theorem?

$$
[\forall x \in S: P(x)] \vee[\forall x \in S: \neg P(x)]
$$

- What is an example of a predicate for which the statement is false? is true?


## practice question

- Exactly one of the following two propositions is a theorem. Which one?
(1) $[\forall x \in S: P(x) \vee Q(x)] \Leftrightarrow[\forall x \in S: P(x)] \vee[\forall x \in S: Q(x)]$
(2) $[\exists x \in S: P(x) \vee Q(x)] \Leftrightarrow[\exists x \in S: P(x)] \vee[\exists x \in S: Q(x)]$
- Justify your answer

