## csci54 -discrete math \& functional programming basic data types: sets, function, relations


"Connecting Discrete Mathematics and Computer Science"
by David Liben-Nowell
https://cs.carleton.edu/faculty/dln/book/

- Python has 4 built-in data types for storing collections of data.
- What are they? How are they different? What is each one good for?
- lists: ordered, indexed, mutable, allows duplicate values
- tuples: ordered, indexed, immutable, allows duplicate values
- dictionaries: unordered, mutable, cannot have duplicate keys
- sets: unordered, unindexed, cannot have duplicate keys, can add/remove elements but can't change existing elements


## Mathematical sets

- A set is an unordered collection of objects
- Given a set $S$ and an object $o$, either $o \in S$ or $o \notin S$
- The cardinality of a set is written $|\mathrm{S}|$ and is the number of elements in the set
- Examples of sets we've seen:
- Int
- Integer
- Char
- Bool
$Z$ : set of integers
$Z^{+:}$set of positive integers
N : set of non-negative integers
Q : set of rationals
R : set of reals


## Defining a mathematical set

- exhaustive enumeration: list everything

$$
S=\{1,2,17\}
$$

- set abstraction: define a set using set operations or a "set builder notation" like our list comprehensions
$Z$ : set of integers
$Z+$ : set of positive integers
N : set of non-negative integers
Q : set of rationals
R : set of reals
The empty set, which contains no elements: \{\} or $\varnothing$

The universal set, $U$

## Set operations (what can you do with sets S and T ?)

- Informally . . .
- S or S': set complement
vet of elements that are in U (the universal set) but not in S
- SUT: set union
- set of elements that are in S or in T
- $\mathrm{S} \cap \mathrm{T}$ : set intersection
" set of elements that are in S and in T
- S-T: set difference
- set of elements that are in S and not in T


## Set operations in set notation

- Sc: set complement
- set of elements that are in $U$ (the universal set) but not in $S$
- $S^{c}=\{x \in U: x \in S\}$
- SUT: set union
- set of elements that are in S or in $T$
- $\mathrm{ST}\{\mathrm{x}: \mathrm{x} \in \mathrm{S}$ or $\mathrm{x} \in \mathrm{T}\}$
$U=\mathbb{Z}^{+}$
$A=\{n: n \geq 6\}$
$B=\{1,2,4,5,7,8\}$
- $\mathrm{S} \cap$ T: set intersection
- set of elements that are in $S$ and in $T$
$A^{C}$
- ST $\{x: x \in S$ and $x \in T\}$
- S-T: set difference
- set of elements that are in S and not in T
- ST $\{x: x \in S$ and $x \notin T\}$


## What can you say about sets S and T ?

- = : set equality
- S and T contain the same elements
- $\subseteq$ : subset
- S contains T
$U=\mathbb{Z}^{+}$
- $\subset$ : proper subset
- S contains T and S does not equal T
- $\supseteq$ : superset
- T contains S
- $~$ : proper superset
- T contains S and T does not equal S
$B=\{1,2,4,5,7,8\}$
- Are either A or B a subset of the other?
- Given an example of a proper superset of B.


## Functions

## Definition 2.46: Function.

Let $A$ and $B$ be sets. A function from $A$ to $B$, written $f: A \rightarrow B$, assigns to each input value $a \in A$ a unique output value $b \in B$; the unique value $b$ assigned to $a$ is denoted by $f(a)$. We sometimes say that $f$ maps a to $f(a)$.

- Example(s) of function(s) from $\{1,2,3\}$ to $\{2,4,6\}$
- What is an example of a function


## Defining functions

- symbolically
- exhaustively
- how would you define the function for "and"?
- what does it map from/to?


## Cartesian product

- The Cartesian product of two sets is written AxB and is defined as:

$$
A x B=\{(x, y): x \in A \text { and } y \in B\}
$$

- What is $A x B$ if $A=\{1,2\}$ and $B=\{$ true, false $\}$ ?
- How would you define the function for "and"?
- How would you define a function which takes two real numbers and returns their average?


## Definitions related to functions

- Given a function
- the domain is the set A
- the co-domain is the set B
- the range (or the image) is the subset of $B$ that are actually mapped to by an element in A.
- Examples:
- IsEven
- Pow (haskell ^)

- what's an example of a function whose domain, co-domain, and range are all the same?


## classifying functions

- one-to-one: a function is one-to-one if, for every element of the codomain, at most one element of the domain maps to it.
- onto: a function is onto if, for every
 element of the co-domain, there is an element of the domain that maps to it.
- alternatively, a function is onto if the co-domain equals the range
- bijection: a function is a bijection if
- it is both one-to-one and onto


## a little on Latex

```
\documentclass{article}
\usepackage{amsmath,amssymb}
\begin{document}
Hello world!
Let's define a few sets
\begin{itemize}
\item Here's a set with one element.
\[ S = \{x \in \mathbb{Z} : x+10=100 \}
\]
\item Here's a set with an infinite
number of elements: $S^C$
\end{itemize}
\end{document}
```

Hello world!
Let's define a few sets

- Here's a set with one element:

$$
S=\{x \in \mathbb{Z}: x+10=100\}
$$

- Here's a set with an infinite number of elements: $S^{C}$
- note: environments, math mode, packages
- a quick intro:
https://www.overleaf.com/learn/latex/Learn_LaTeX_in_30_min

