Lecture 22: Analyzing Algorithms

CS 51P

November 30, 2023

Sorting Algorithms

Selection Sort

```
def selection_sort(l):

  # for each pos in list
  for pos in range(len(l)):

    # find obj
    min_index =
    for i in ra
      if l[i] <
        min_ind

    # swap that
    (l[pos], l[
        (l[min_
```

Insertion Sort

```
def insertion_sort(l):

  # for each obj in list
  for pos in range(len(l)):

    rect position

    0 and
    [curr_pos-1]:
    l[curr_pos-1])=\
    -1],l[curr_pos])
    _pos - 1
```

Merge Sort

```
def merge_sort_helper(lst, start, end):
    # Base Case
    if end-start < 2:
        return
    # Recursive Case
    else:
        middle = start + int((end-start) / 2)
        le)
        merge(lst, start, end)
```

Which algorithm is better?

What Makes a Good Algorithm?

Suppose you have two possible algorithms that do the same thing; which is *better*?

What do we mean by *better*?

- Correct(er)?
- Faster?
- Less space?
- Less power consumption?
- Easier to code?
- Easier to maintain?
- Required for homework?

Basic Step: one “constant time” operation

Constant time operation: its time doesn’t depend on the size or length of anything. Always roughly the same. Time is bounded above by some number

Example Basic steps:

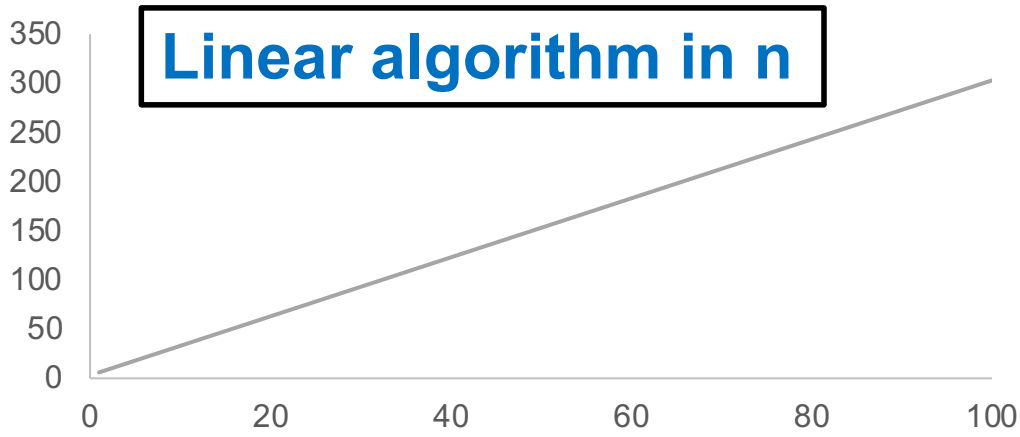
- Access value of a variable, list element, or object attr
- Assign to a variable, list element, or object attr
- Do one arithmetic or logical operation
- Call a function

Counting Steps

```
# Store sum of 0..n-1 in sum
sum = 0
for i in range(n):
    sum = sum + i
```

Statement:	# times done
sum = 0	1
i= v	n
sum = sum + i	n
Total steps:	2n + 1

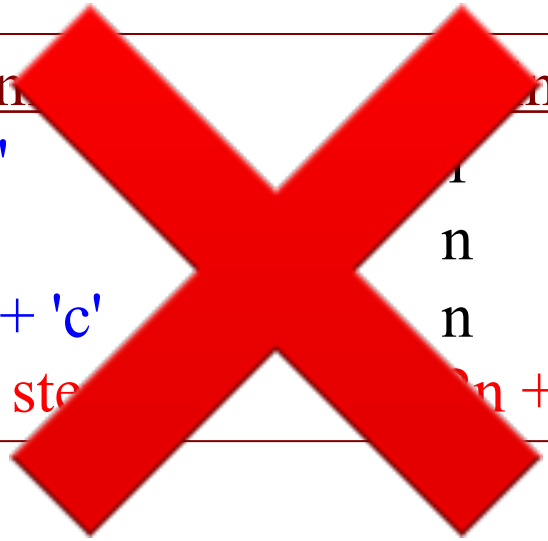
All basic steps take time 1. There are n loop iterations. Therefore, takes time proportional to n.



Not all operations are basic steps

```
# Store n copies of ‘c’ in s
s = ""
for i in range(n):
    s = s + 'c'
```

Staten	mes done
s = ""	1
i = v	n
s = s + 'c'	n
Total ste	n + 1



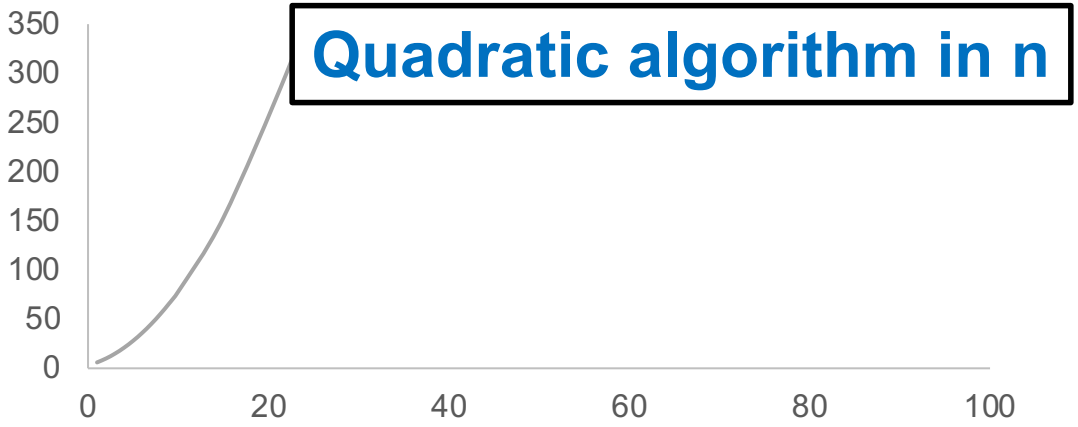
Concatenation is not a basic step. For each i, concatenation creates and fills a sequence with i elements.

Not all operations are basic steps

```
# Store n copies of ‘c’ in s
s = ""
for i in range(n):
    s = s + 'c'
```

Statement:	# times	# steps
s = ""	1	1
i = v	n	1
s = s + 'c';	n	i
Total steps:	(n-1)*n/2 + n + 1	

Concatenation is not a basic step. For each i, concatenation creates and fills a sequence with i elements.



Quadratic algorithm in n

Linear versus quadractic

```
# Store sum of 1..n in sum
sum = 0
for i in range(1, n+1):
    sum = sum + k;
```

Linear algorithm

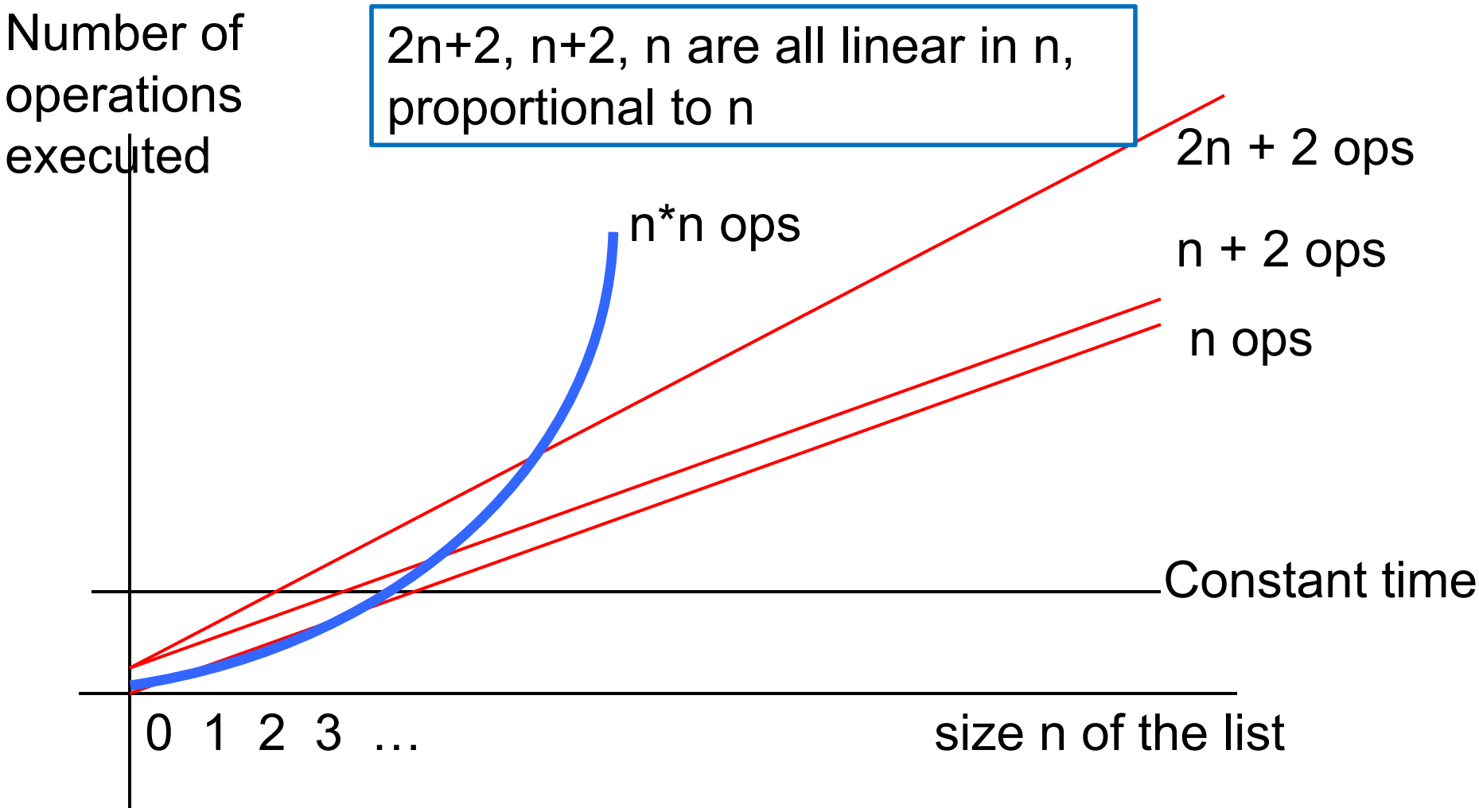
```
# Store n copies of ‘c’ in s
s = ""
for i in range(n):
    s = s + 'c'
```

Quadratic algorithm

In comparing the runtimes of these algorithms, the exact number of basic steps is not important. What’s important is that

One is linear in n—takes time proportional to n
One is quadratic in n—takes time proportional to n^2

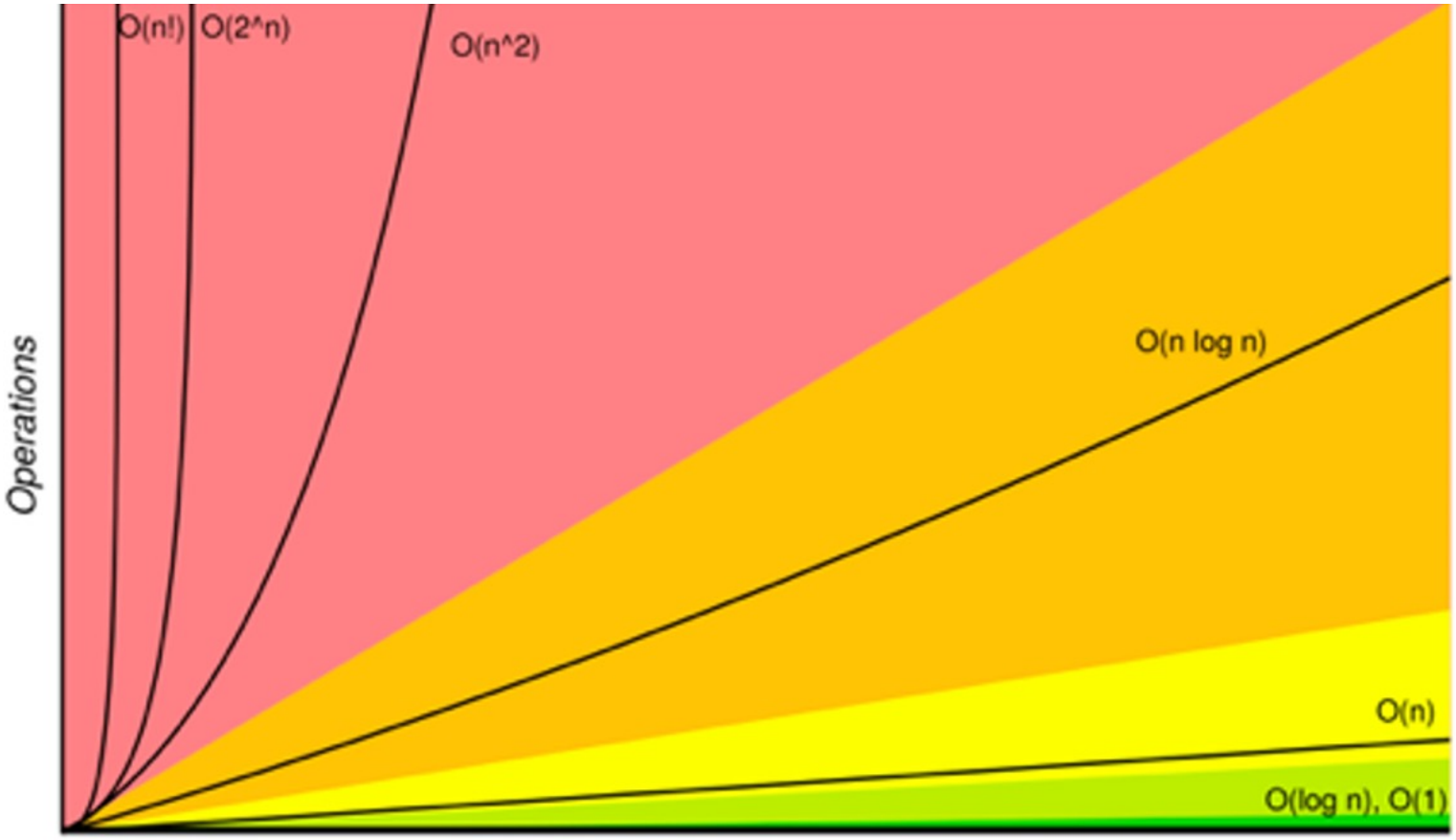
Looking at execution speed



"Big O" Notation

- $n^2 + 2n + 5$ $O(n^2)$
- $1000n + 25000$ $O(n)$
- $\frac{2^n}{15} + n^{100}$ $O(2^n)$
- $n \log n + 25n$ $O(n \log n)$

Order of growth



How Fast is Fast enough?

$O(1)$	constant	excellent
$O(\log n)$	logarithmic	excellent
$O(n)$	linear	good
$O(n \log n)$	n log n	pretty good
$O(n^2)$	quadratic	maybe OK
$O(n^3)$	cubic	not good
$O(2^n)$	exponential	too slow
$O(n!)$	factorial	too slow

Evaluating Speed of Selection Sort

```
def selection_sort(l):
    # for each pos in the list
    for pos in range(len(l)):
        # find the object that should be there
        min_index = pos
        for i in range(pos+1, len(l)):
            if l[i] < l[min_index]:
                min_index = i

        # swap the object to position pos
        (l[pos], l[min_index]) = \
          (l[min_index], l[pos])
```

# Times	# Steps
n	O(1)
n	O(1)
n*O(n)	O(1)
n*O(n)	O(1)
<= n*O(n)	O(1)
n	O(1)

Selection Sort runs in time $O(n^2)$

Comparison

	selection sort
worst case	$O(n^2)$
best case	$O(n^2)$
avg case	$O(n^2)$
space	$O(1)$

Evaluating Speed of Insertion Sort

```
def insertion_sort(l):
    # for each obj at position pos
    for pos in range(1, len(l)):


        # move the obj to the right position
        curr_pos = pos
        while curr_pos > 0 and \
                l[curr_pos] < l[curr_pos-1]:
            (l[curr_pos], l[curr_pos-1]) = \
               (l[curr_pos-1], l[curr_pos])
            curr_pos -= 1
```

# Times	# Steps
n	O(1)
n	O(1)
<=n*O(n)	O(1)
<= n*O(n)	O(1)
<= n*O(n)	O(1)

Insertion Sort runs in time $O(n^2)$

Comparison

	selection sort	insertion sort
worst case	$O(n^2)$	$O(n^2)$
best case	$O(n^2)$	$O(n)$
avg case	$O(n^2)$	$O(n^2)$
space	$O(1)$	$O(1)$

Evaluating Speed of Merge Sort

```
def merge_sort_helper(lst, start, end):
  # Base Case
  if (end-start) < 2:
    return
  # Recursive Case
  else:
    middle = start + int((end-start)/2)
    merge_sort_helper(lst, start, middle)
    merge_sort_helper(lst, middle, end)
    merge(lst, start, end)

def merge_sort(lst):
  merge_sort_helper(lst, 0, len(lst))
```

# Times	# Steps
1	O(1)
<=1	O(1)
1	O(1)
	?
	?
	?

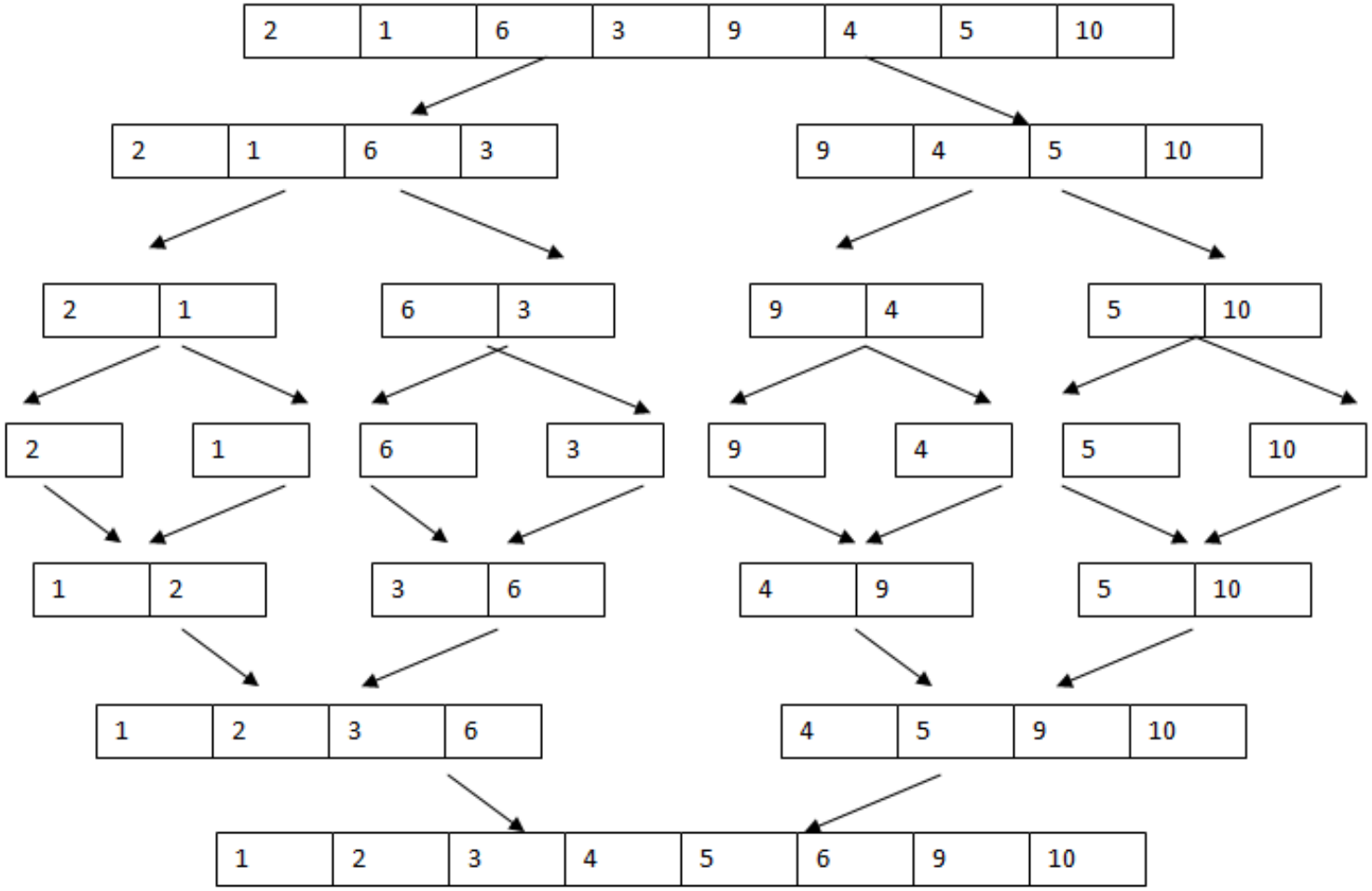
Evaluating Speed of Merge Sort

```
def merge(lst, start, end):
  middle = (end-start)//2
  first = lst[:middle]
  pos, i, j = start, start, middle
  while i < middle and j < end:
    if first[i] < lst[j]:
        lst[pos] = first[i]
        i += 1
    else:
        lst[pos] = lst[j]
        j += 1
    pos += 1
  if j == end:
    while i < middle:
        lst[pos] = first[i]
        pos +=
        i += 1
```

# Times	# Steps
1	O(1)
1	O(end-start)
1	3
(end-start)/2	O(1)
(end-start)/2	O(1)
<=(end-start)/2	O(1)
<=(end-start)/2	O(1)
<=(end-start)/2	O(1)
<=(end-start)/2	O(1)
(end-start)/2	O(1)
1	1
<=(end-start)/2	O(1)
<=(end-start)/2	O(1)
-start)/2	O(1)
-start)/2	O(1)

Merge runs in time $O(n)$

Evaluating Speed of Merge Sort



Merge is executed for *O(log(n))* times

Comparison

	selection sort	insertion sort	merge sort
worst case	$O(n^2)$	$O(n^2)$	O(n log n)
best case	$O(n^2)$	O(n)	O(n log n)
avg case	$O(n^2)$	$O(n^2)$	O(n log n)
space	O(1)	O(1)	O(n)

Sorting in Python

- List.sort()
  - Sorts list in place
  - Optional argument reverse=True to reverse order (greatest->least)

- sorted(lst)
  - Creates sorted copy of list
  - Optional arguments reverse