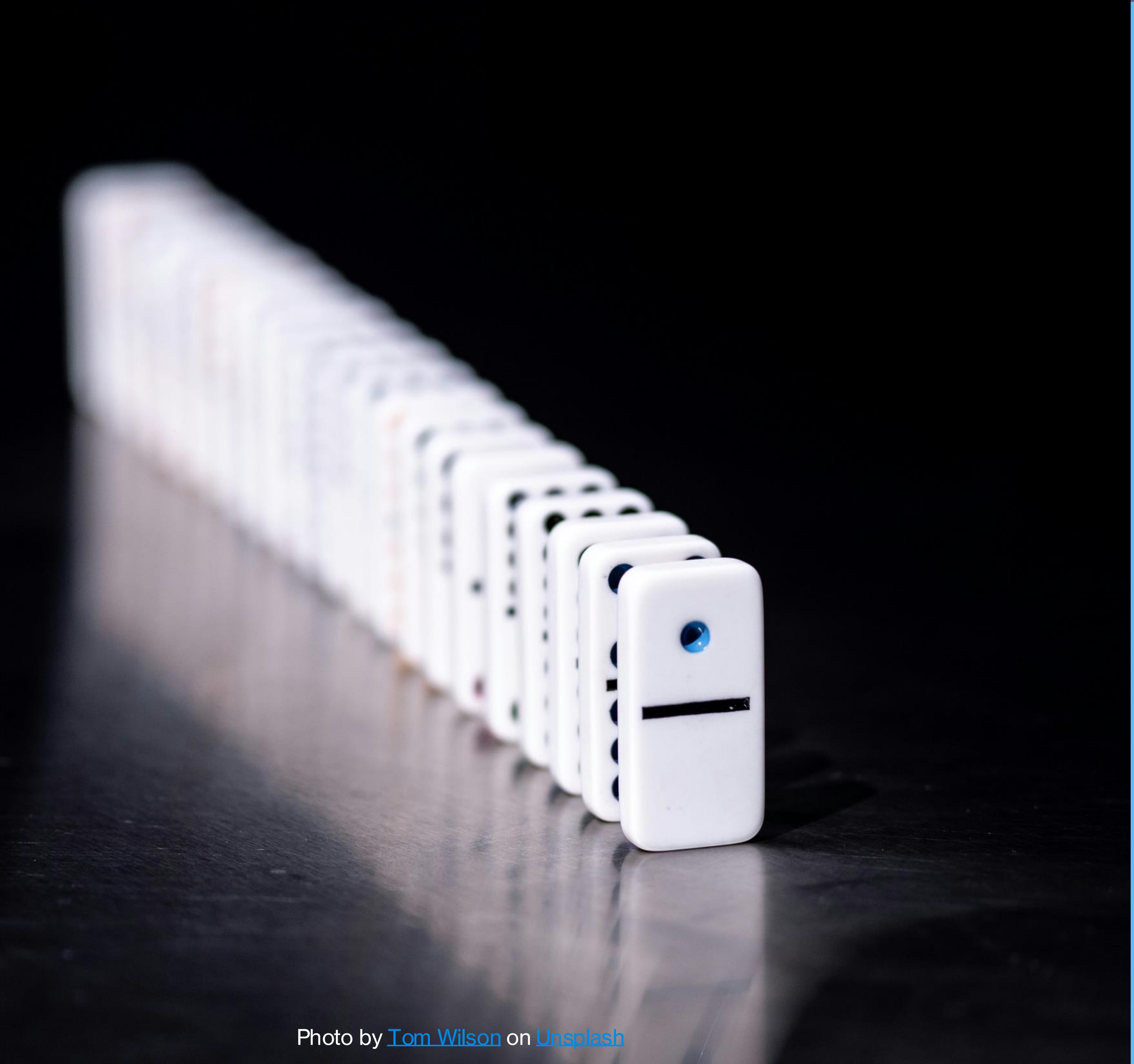

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Mathematical Foundations

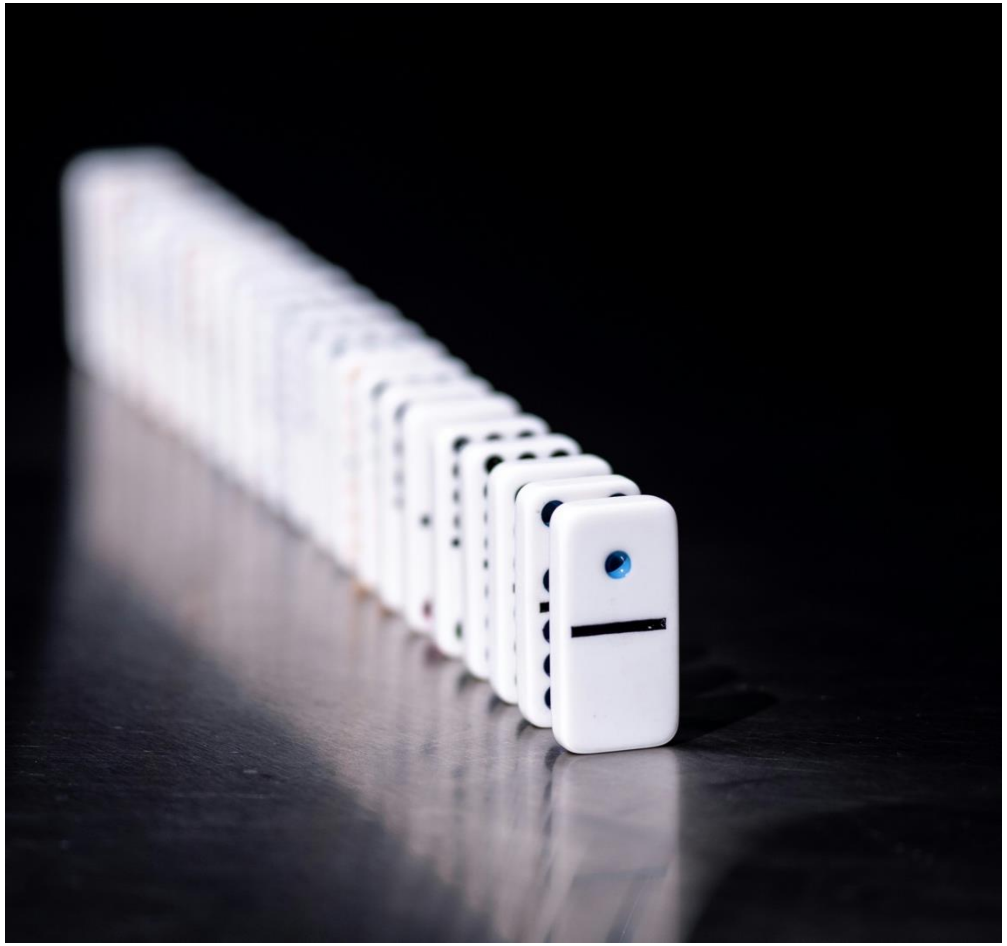
Proof by induction

CS51 – Spring 2026

Proof by induction

- To prove that a property $P(n)$ holds for all non-negative integers n, we prove that:
  - $P(0)$ is true,
  - for every $n \geq 0$, if $P(n)$ is true, then $P(n+1)$ is true, too.
- The proof of $P(0)$ is called the **base case**.
- The proof that $P(n) \Rightarrow P(n+1)$ is called the **inductive case**.
- Practically, the principle of mathematical induction says the following: to prove that a statement $P(n)$ is true for all non-negative integers n, we can prove that P "starts being true" (the base case) and that P "never stops being true" (the inductive case).

Proof by induction analogies



- We have an infinitely long line of dominoes, numbered $0, 1, 2, \ldots, n, \ldots$
- To convince someone that the n -th domino falls over, you can convince them that:
  - the 0-th domino falls over, and
  - whenever one domino falls over, the next domino falls over too. (One domino falls, and they keep on falling. Thus, for every $n \geq 0$, the $n + 1$ -th domino falls, if we assume that the n-th domino falls.)
- Similar analogies with climbing a ladder or whispering a secret down an alley.

An aside: summation notation

- Let $x_1, x_2, \ldots, x_n$ be a sequence of n numbers. We write $\sum_{i=1}^{n} x_i$ to denote the sum of these n numbers.
- That is $\sum_{i=1}^{n} x_i = x_1 + x_2 + \cdots + x_n$
- We read this as the sum for i equals 1 to n of x_i.
- i is called the index of summation or the index variable.
- There is a correspondence with for-loops like:
  - ```
    result=0
    for i in range(1,n+1):
          result += x_n
    return result
    ```
- Note that $\sum_{i=1}^{0} x_i = 0$: when you add nothing together, you end up with zero.
    ```

Examples: summation notation

- Let $a_1 = 2, a_2 = 4, a_3 = 8, a_4 = 16$. Then:
  - $\sum_{i=2}^{4} a_i$
    - $\sum_{i=2}^{4} a_i = 4 + 8 + 16 = 28$
  - $\sum_{i=1}^{3} (a_i + 1)$
    - $\sum_{i=1}^{3} (a_i + 1) = (2 + 1) + (4 + 1) + (8 + 1) = 17$
  - $\sum_{i=1}^{4} i$
    - $\sum_{i=1}^{4} i = 1 + 2 + 3 + 4 = 10$

Practice time: summation notation

- Let $a_1 = 2, a_2 = 4, a_3 = 8, a_4 = 16$. Evaluate the following expressions:
  - $\sum_{i=1}^{4} i^2$
  - $\sum_{i=2}^{4} i^2$
  - $\sum_{i=1}^{4}(a_i+i^2)$
  - $\sum_{i=1}^{4} 5$

Answer: summation notation

- Let $a_1 = 2, a_2 = 4, a_3 = 8, a_4 = 16$. Evaluate the following expressions:
  - $\sum_{i=1}^{4} i^2 = 1^2 + 2^2 + 3^2 + 4^2 = 30$
  - $\sum_{i=2}^{4} i^2 = 2^2 + 3^2 + 4^2 = 29$
  - $\sum_{i=1}^{4} (a_i + i^2) = (2 + 1^2) + (4 + 2^2) + (8 + 3^2) + (16 + 4^2) = 60$
  - $\sum_{i=1}^{4} 5 = 5 + 5 + 5 + 5 = 20$

Example: sum of powers of two

- Using mathematical induction, prove that for any non-negative integer n, we have that $\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$.
- Let's start with a plausibility check to test the formula for small values of n.
  - When $n = 1$: $\sum_{i=0}^{1} 2^i = 2^0 + 2^1 = 3$. $2^{n+1} - 1 = 2^2 - 1 = 3$.
  - When $n = 2$: $\sum_{i=0}^{2} 2^i = 2^0 + 2^1 + 2^2 = 7$. $2^{n+1} - 1 = 2^3 - 1 = 7$.
  - When $n = 3$: $\sum_{i=0}^{3} 2^i = 2^0 + 2^1 + 2^2 + 2^3 = 15$. $2^{n+1} - 1 = 2^4 - 1 = 15$.
- These small claims all check out, so it's reasonable to try to prove the claim.

Example: Induction to prove sum of powers of two

- Let $P(n)$ denote the property $\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$. We will prove that for every non-negative integer, $P(n)$ is true by induction on n.

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- Let $P(n)$ denote the property $\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$. We will prove that for every non-negative integer, $P(n)$ is true by induction on n.
- *Base case* $(n = 0)$: We must prove $P(0)$, that is, $\sum_{i=0}^{0} 2^i = 2^{0+1} - 1$.
  This fact is easy to prove, because both sides are equal to 1:
  - $\sum_{i=0}^{0} 2^i = 2^0 = 1$
  - $2^{0+1} - 1 = 2 - 1 = 1$

Example: Induction to prove sum of powers of two

- *Inductive case* $(n \geq 0)$: We must prove that if $P(n) \Longrightarrow P(n+1)$, for an arbitrary $n \geq 0$.
- We prove this by assuming the antecedent/hypothesis (we assume $P(n)$ and prove $P(n+1)$).
- The assumption $P(n)$ is $\sum_{i=0}^{n} 2^i = 2^{n+1} - 1$. (*)

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- We can now prove $P(n+1)$ (under the assumption (*)) by showing that the left- and right-hand sides of $P(n+1)$ are equal, that is, we want to prove that $\sum_{i=0}^{n+1} 2^i = 2^{(n+1)+1} - 1$:

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- $\sum_{i=0}^{n+1} 2^i = \left[\sum_{i=0}^{n} 2^i\right] + 2^{n+1}$ *by the definition of summation*

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  $= [2^{n+1} - 1] + 2^{n+1}$ *by (*)*

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  $= [2^{n+1} - 1] + 2^{n+1}$ *by (*)*

  $= 2^{n+1} - 1 + 2^{\mathrm{n}+1} = 2 \times 2^{n+1} - 1 = 2^{(n+1)+1} - 1.$ *by algebraic manipulation*

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- $\sum_{i=0}^{n+1} 2^i = \left[\sum_{i=0}^{n} 2^i\right] + 2^{n+1}$ *by the definition of summation*
  $= [2^{n+1} - 1] + 2^{n+1}$ *by (*)*
  $= 2^{n+1} - 1 + 2^{\mathrm{n}+1} = 2 \times 2^{n+1} - 1 = 2^{(n+1)+1} - 1.$ *by algebraic manipulation*
- We have thus shown that $\sum_{i=0}^{n+1} 2^i = 2^{(n+1)+1} - 1$, in other words we have proven $P(n+1)$.
- By the principle of mathematical induction, we have shown $P(n)$ holds for all integers $n \geq 0$.

Template for proofs by induction

1. Clearly state the claim to be proven. Clearly state that the proof will be by induction, and clearly state the variable upon which induction will be performed.
2. State and prove the base case.
3. State and prove the inductive case. Within the statement and proof of inductive case:
   a) state the inductive hypothesis
   b) state what we need to prove
   c) Prove it, making use of the inductive hypothesis and stating where it was used.

Example: Induction to prove sum of powers of -1

- 1. Claim: For any integer $n \geq 0$, we have that $\sum_{i=0}^{n}(-1)^{\,i} = \begin{cases} 1, & if\ n\ is\ even \\ 0, & if\ n\ is\ odd. \end{cases}$

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- Let $P(n)$ denote the property $\sum_{i=0}^{n}(-1)^{\,i} = \begin{cases} 1, & if\ n\ is\ even \\ 0, & if\ n\ is\ odd. \end{cases}$. We will prove that for every non-negative integer, $P(n)$ is true by induction on n.

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- *2. Base case* $(n = 0)$: We must prove $P(0)$. Indeed, $\sum_{i=0}^{0}(-1)^{\,i} = (-1)^0 = 1$ and 0 is even.

Example: Induction to prove sum of powers of -1

- *3.a. Inductive case* $(n \geq 0)$: We assume the inductive hypothesis $P(n)$ for an arbitrary $n \geq 0$, namely:
- $\sum_{i=0}^{n}(-1)^{\,i} = \begin{cases} 1, & if\ n\ is\ even \\ 0, & if\ n\ is\ odd \end{cases}$ (*).

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- c. $\sum_{i=0}^{n+1}(-1)^{\,i} = \left[\sum_{i=0}^{n}(-1)^{\,i}\right] + (-1)^{(n+1)}$ *by the definition of summation*

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$$= \begin{cases} 1 + (-1)^{(n+1)}, & if\ n\ is\ even \\ 0 + (-1)^{(n+1)}, & if\ n\ is\ odd \end{cases}$$ *by using inductive hypothesis(*)*

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n *is odd* $\Leftrightarrow$ $n+1$ *is even*

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$= \begin{cases} 0, & if\ n+1\ is\ odd \\ 1, & if\ n+1\ is\ even \end{cases}$ Thus, we have shown $P(n)$ holds for all integers $n \geq 0$.

Practice time

- You are given the following theorem about the sum of the first n non-negative integers:
- For any integer $n \geq 0$, we have that $\sum_{i=0}^{n} i = 0 + 1 + 2 + \cdots + \mathrm{n} = \frac{n(n+1)}{2}$.
  - For example, for $n = 4$, we have $0 + 1 + 2 + 3 + 4 = 10 = \frac{4(4+1)}{2}$
- Using the principle of mathematic induction and the 3 steps of the template we just saw, prove it.

- This summation is not an arbitrary arithmetic example. You will encounter it again and again when analyzing the time complexity of fundamental algorithms. More to come…

Answer

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- *2. Base case* $(n = 0)$: We must prove $P(0)$. Indeed, $\sum_{i=0}^{0} i = 0$ and $\frac{0(0+1)}{2} = 0$, too. Thus $P(0)$ holds.

Answer

- *3.a. Inductive case* $(n \geq 0)$: We assume the inductive hypothesis $P(n)$ for an arbitrary $n \geq 0$, namely:
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$= \frac{n(n+1)}{2} + \frac{2(n+1)}{2} = \frac{n(n+1)+2(n+1)}{2} = \frac{(n+1)(n+2)}{2} = \frac{(n+1)((n+1)+1)}{2}$ *using algebraic manipulations*

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Thus, $P(n+1)$ is true and through the principle of mathematical induction we have shown $P(n)$ holds for all integers $n \geq 0$.

Loop invariants and induction

- The parallels between loop invariants and induction are pretty direct.
- A *loop invariant* for a loop L is a logical property P such that:
  - i) P is true before L is first executed; and
  - (ii) if P is true at the beginning of an iteration of L, then P is true after that iteration of L. that is true at the beginning and end of every iteration of a loop.

Property i corresponds to the base case and property ii to the inductive case.