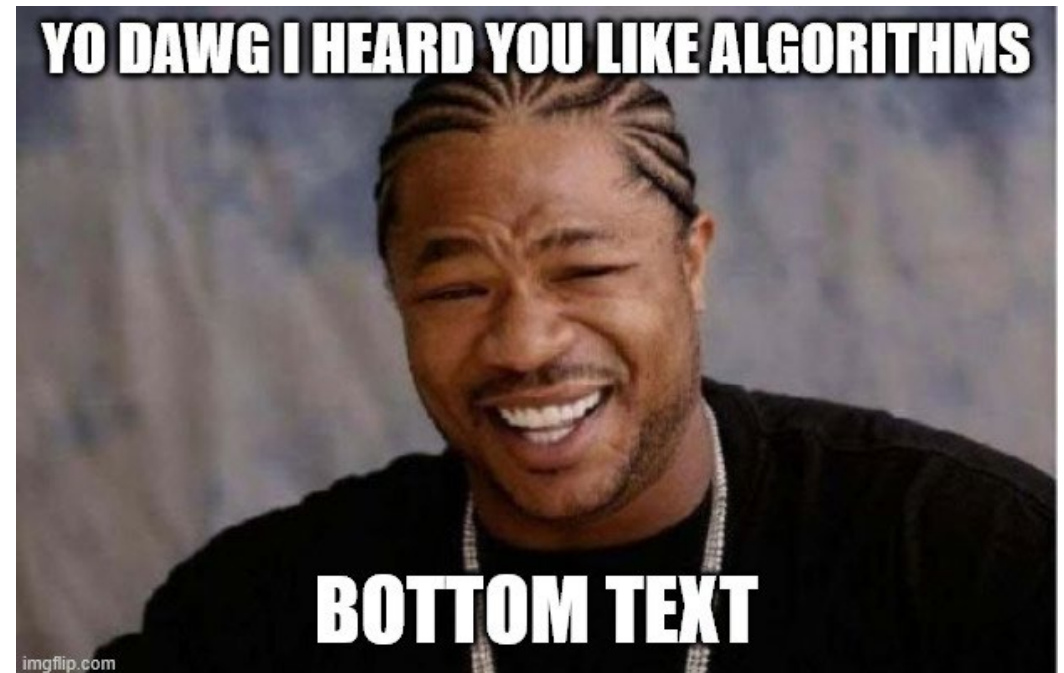


CS 181 – Advanced Algorithms

Welcome back to campus
and to this class!



Who am I

- Prof. Michael Zlatin. I often go by Mik.
- Ph.D. from Carnegie Mellon in Pittsburgh, Pennsylvania. Still getting used to this coast.
- I like all sports, currently volleyball and rock climbing.
- Etc. etc.



Hiking in Switzerland. Me (left), cow (right).

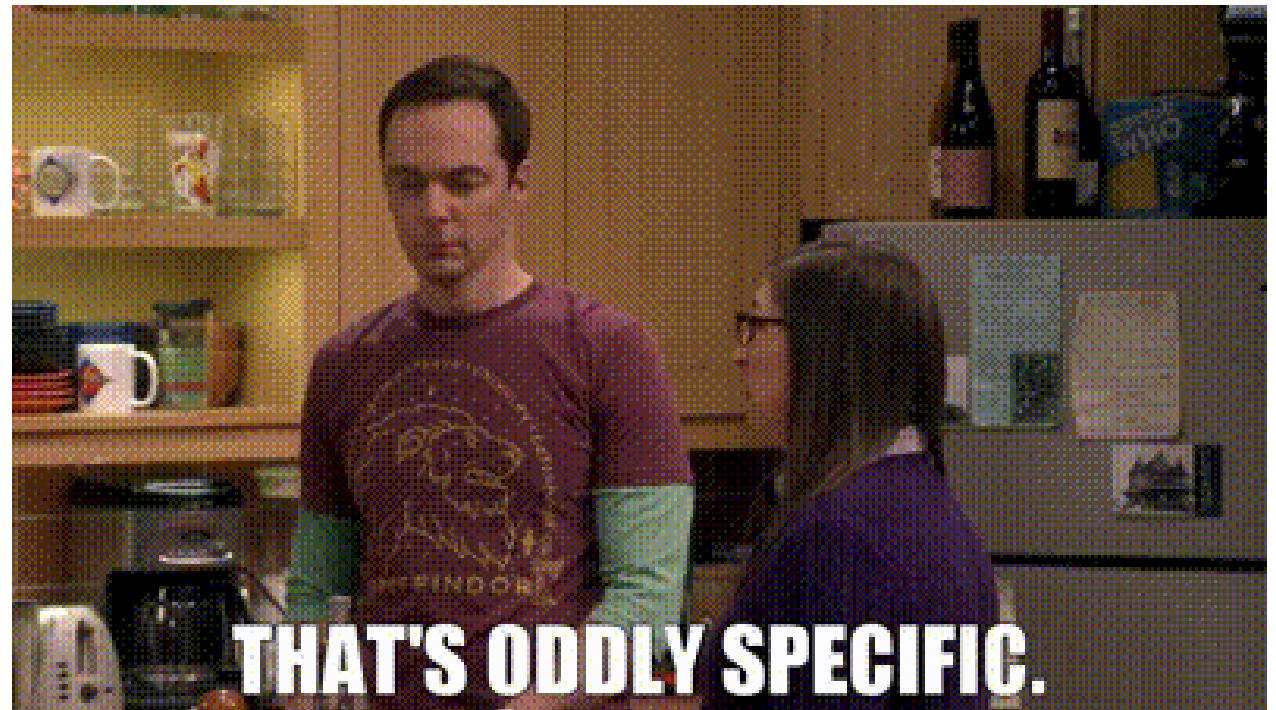
Algorithms . . .

Problems:

- Sorting a list of numbers
- Graph traversal
- Shortest way to get from node a to node b in a graph
- Minimum spanning trees
- Flows?

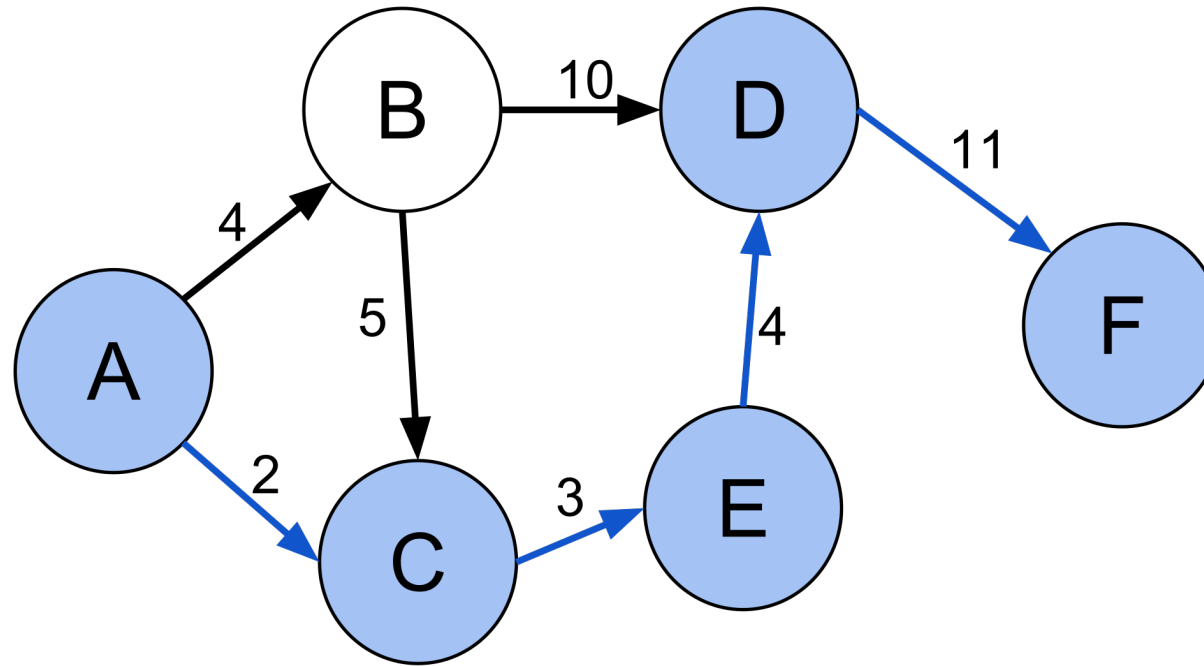
Solutions:

- Bubble sort, merge sort
- Breadth First Search, Depth First Search
- Dijkstra's algorithm, Bellman-Ford, FW . . .
- Prim's, Kruskal's
- Ford-Fulkerson, Edmonds-Karp



Algorithmic
Paradigms

An algorithm is an answer to a class of problems



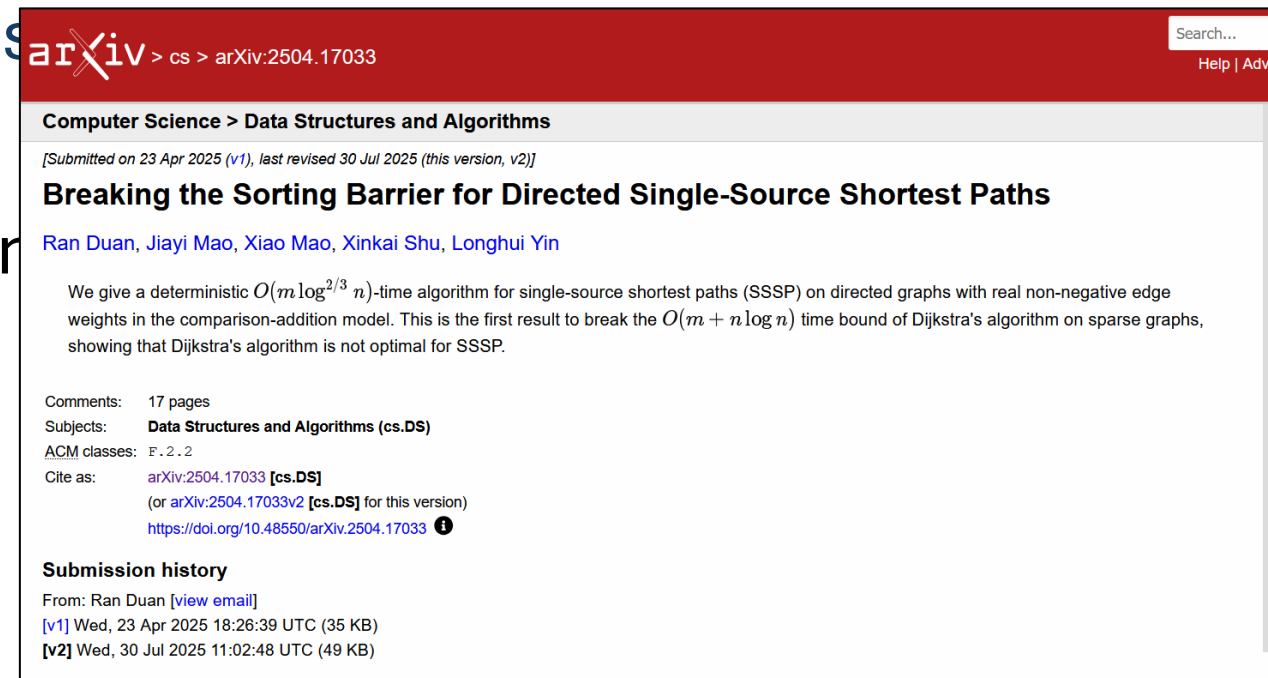
What is the length of the shortest path from A to F in this graph?

How do we compare algorithms?

- **Feasibility:** always outputs a **valid** path from s to t in the graph
- **Optimality:** The path is always a **shortest** path
- **Running time:** on a graph with n nodes and m edges, the algorithm runs in $O(m + n \log n)$ time.

This year: “improvement” on Dijkstra’s: $O(m \cdot \log^{2/3} n)$.

Best Paper Award at STOC 2025



The screenshot shows the arXiv page for the paper "Breaking the Sorting Barrier for Directed Single-Source Shortest Paths" by Ran Duan, Jiayi Mao, Xiao Mao, Xinkai Shu, and Longhui Yin. The page is categorized under Computer Science > Data Structures and Algorithms. It includes submission details, a brief abstract, and a submission history.

arXiv > cs > arXiv:2504.17033

Search... Help | Adv

Computer Science > Data Structures and Algorithms

[Submitted on 23 Apr 2025 (v1), last revised 30 Jul 2025 (this version, v2)]

Breaking the Sorting Barrier for Directed Single-Source Shortest Paths

Ran Duan, Jiayi Mao, Xiao Mao, Xinkai Shu, Longhui Yin

We give a deterministic $O(m \log^{2/3} n)$ -time algorithm for single-source shortest paths (SSSP) on directed graphs with real non-negative edge weights in the comparison-addition model. This is the first result to break the $O(m + n \log n)$ time bound of Dijkstra's algorithm on sparse graphs, showing that Dijkstra's algorithm is not optimal for SSSP.

Comments: 17 pages
Subjects: Data Structures and Algorithms (cs.DS)
ACM classes: F.2.2
Cite as: arXiv:2504.17033 [cs.DS]
(or arXiv:2504.17033v2 [cs.DS] for this version)
<https://doi.org/10.48550/arXiv.2504.17033>

Submission history

From: Ran Duan [view email]
[v1] Wed, 23 Apr 2025 18:26:39 UTC (35 KB)
[v2] Wed, 30 Jul 2025 11:02:48 UTC (49 KB)

Can we do it faster?

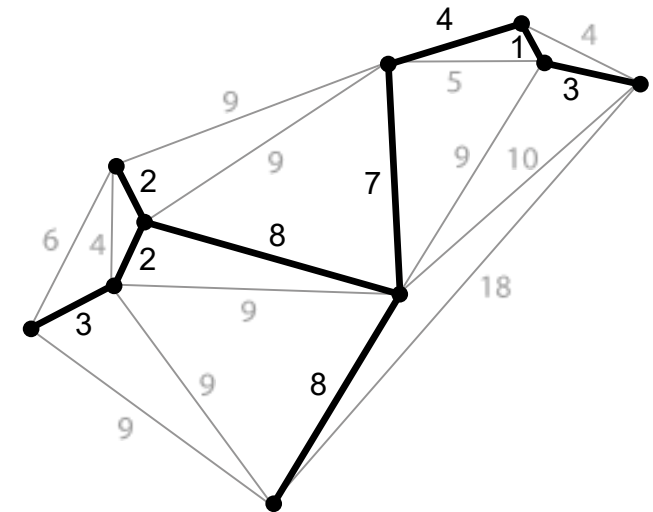
This is a good question

Faster algorithms for fundamental problems:

For example, MST:

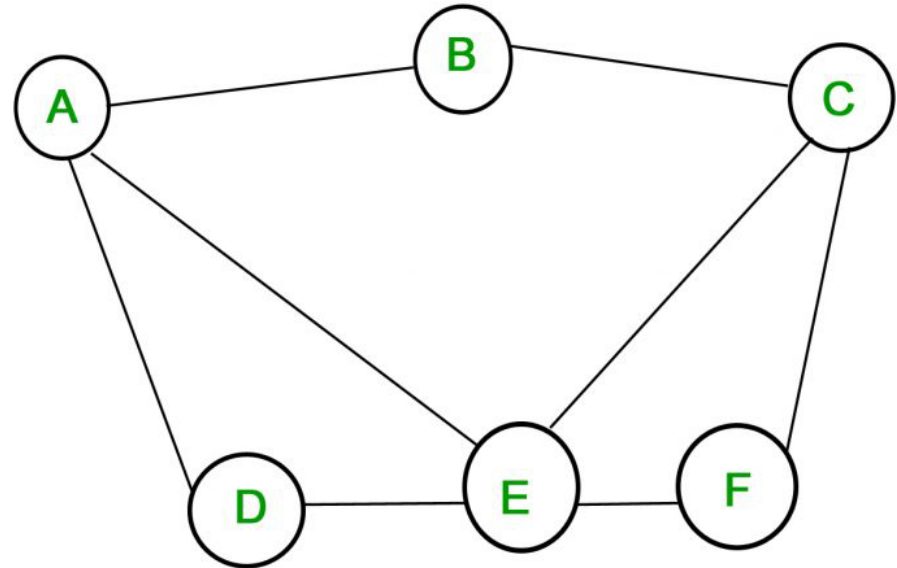
- Naïve brute force: 2^m
- Kruskal [1956]: $O(m \cdot \log m)$
- Chazzele [2000]: $O(m \cdot \alpha(m))$, where α is the inverse Ackerman function.

$$\alpha(2^{2^{2^{2^{16}}}}) \approx 4$$



Time complexity and input size

- Depends on how you store the graph.
- Graph on n nodes with m edges:
 - Adjacency matrix: $O(n^2)$
 - Adjacency list: $O(n + m)$
- We will always just assume it is given as an adjacency list.



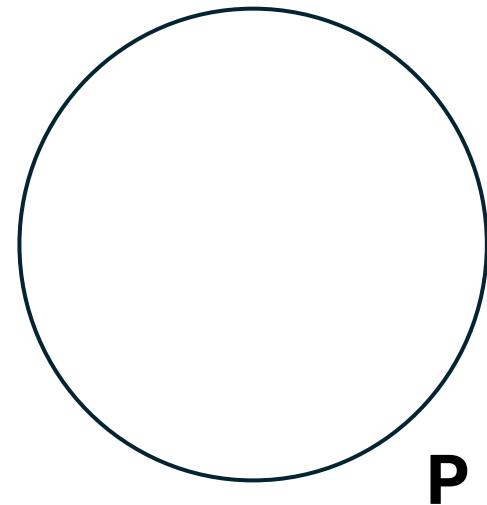
Q: Which problems will we be able to solve in practice?



A working definition: Those with poly-time algorithms

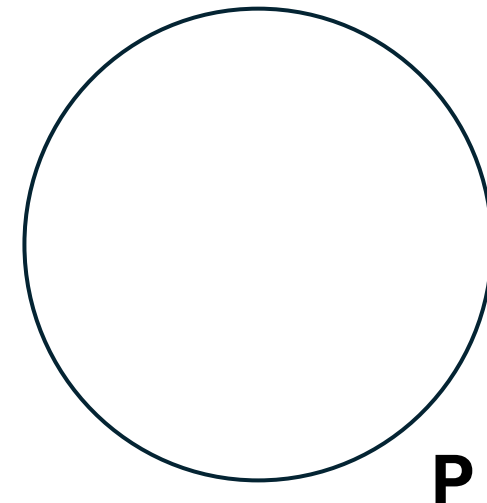
Theory. Definition is broad and robust.

Practice. Poly-time algorithms scale well on larger inputs



Q: Which problems admit polynomial time algorithms?

yes	probably no
shortest path	longest path
min cut	max cut
2-satisfiability	3-satisfiability
planar 4-colorability	planar 3-colorability
bipartite vertex cover	vertex cover
matching	3d-matching
primality testing	factoring
linear programming	integer linear programming



Can we do it faster?

Karp: Either **all** of these are in **P**,
or **none** are:

- SAT
- 3-SAT
- Clique
- Independent Set
- Vertex Cover
- Hamiltonian Cycle
- Subset Sum
- 3D-matching
- Steiner Tree

“It’s all or nothing baby”



Richard Karp

NP-hard: If there is a polynomial time algorithm for an NP-hard problem, then there is a poly time algorithm for all problems in NP.

Course Goals

Questions?

- Goal #1: What are the **outer reaches** of problems in **P**
 - Flows, cuts, matchings, linear programming. More paradigms.
 - **Utilize** this toolbox effectively and **communicate** solutions well
- Goal #2: How do we handle problems for which we believe an efficient algorithm **does not exist**?
 - Fast algorithms for NP-hard problems: approximation and parameterization
 - Problems where the input is not fully known: online and streaming