

Test taking advice

Read the questions carefully!

- Don't spend too much time on any problem
 if you get stuck, move on and come back
- When you finish answering a question, reread the question and make sure that you answered everything the question asked
- Think about how you might be able to reuse an existing algorithm/approach
- Show your work (I can't give you partial credit if I can't figure out what went wrong)
- Don't rely on the book/notes for conceptual things
 Do rely on the notes for a run-time you may not remember, etc.

High-level approaches

Algorithm tools

- Divide and conquer
 assume that we have a solver, but that can only solve subproblems
- define the current problem with respect to smaller problems
 Key: sub-problems should be non-overlapping
- Dynamic programming
- Same as above
- Key difference: sub-problems are overlapping
- Once you have this recursive relationship:
 figure out the data structure to store sub-problem solutions
- tigure out the data structure to store sub-problem solution
 work from bottom up

High-level approaches

Algorithm tools cont.

Greedy

- Same idea: most greedy problems can be solve using dynamic programming (but generally slower)
 Key difference: Can decide between overlapping subproblems without having to calculate them (i.e., we can make
- a local decision)

Min-capacity cut

- Bottleneck edge
- Matching problems
 Numerical maximization/minimization problems

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Data structures

A data structure

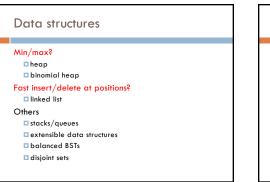
Stores data

Supports access to/questions about data efficiently
 the different bias towards different actions
 No single best data structure

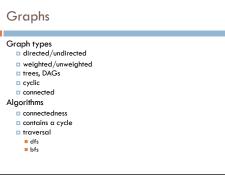
Fast access/lookup?

- If keys are sequential: array
- If keys are non-sequential or non-numerical: hashtable
 Guaranteed run-time/ordered: balanced binary search tree

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Graphs

Graph algorithms cont.

minimum spanning trees (Prim's, Kruskal's)

shortest paths

single source (BFS, Dijskstra's, Bellman-Ford)
 all pairs (Johnson's, Floyd-Warshall)

topological sort

flow

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Other topics...

Analysis tools

recurrences (master method, recurrence trees)
big-O
amortized analysis

NP-completeness

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proving NP-completeness
 reductions

Proofs: general

Be clear and concise

Make sure you state assumptions and justify each step

Make sure when you're done you've shown what you need to show

Proof by induction

- State what you're trying to prove We show that XXX using proof by induction
- 2. Prove base case
- 3. State the inductive hypothesis
- 4. Inductive proof
- State what you want to show (may include a variable change, e.g., k in instead of n)
- Show a step by step derivation from the left hand side resulting in the right hand side. Give justifications for steps that are non-trivial

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Prof by induction: structural

Use induction to prove that the number of degree-2 nodes in a non-empty binary tree is 1 less than the number of leaves. Recall that the degree of a vertex in a tree is the number of children that it has.

- State what you're trying to prove We show that XXX using proof by induction
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 Show a step by step derivation from the left hand side resulting in the right hand side. Give justifications for steps that are nontrivial

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Other (important) places we saw proofs

Recurrences (substitution method)

Big O (needed find constants c n₀)

Greedy algorithm correctness (proof by contradiction or stays ahead—induction —)

Proof of algorithm correctness (MSTs, Flow)

NP-completeness (proving correctness of reductions)

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Recurrences

- Three ways to solve:
- Substitution
- Recurrence tree (may still have to use substitution to verify)
- Master method

Recurrences

T(n) = 2T(n/3) + d

T(n) = aT(n/b) + f(n)

$$\begin{split} & \text{if } f(n) = O(n^{\log_3 \sigma \cdot c}) \text{ for } \varepsilon > 0, \text{ then } T(n) = \Theta(n^{\log_3 \sigma}) \\ & \text{if } f(n) = \Theta(n^{\log_3 \sigma}), \text{ then } T(n) = \Theta(n^{\log_3 \sigma} \log n) \\ & \text{if } f(n) = \Omega(n^{\log_3 \sigma \cdot c}) \text{ for } \varepsilon > 0 \text{ and } af(n/b) \le cf(n) \text{ for } c < 1 \\ & \text{ then } T(n) = \Theta(f(n)) \end{split}$$

 $T(n) = T(n-1) + \log n$

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Dynamic programming

Method for solving problems where optimal solutions can be defined in terms of optimal solutions to subproblems

AND

the subproblems are overlapping

Local decisions result in *different* subproblems. Not obvious how to make the first choice.

DP advice

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Write the recursive definition

What is the input/output to the problem? What would a solution look like? What are the options for picking the first component of a solution? Assume you have a solver for subproblems. How can you combine the first decision with answer to subproblem.

Define DP structure: what are subproblems indexed by?

State how to fill in the table (including base cases and where the answer is)