Announcements

Today is the last regular lecture. There will be no classes Friday and Monday so you can prepare your final project for presentation.

Presentations will be Wednesday and Friday of next week, and the following Monday. The last day of classes we will meet to pull together the various streams of the course and to do course evaluations. The project is due at the beginning of class on Wednesday.

As stated in the class e-mail, there will be no final exam.

CS thesis presentations will be on Thursday late afternoon (starting from 4:15 until 5:45 p.m. and Friday from 1:30 p.m. to 3:45 p.m.

Classifying quantifiers

We can classify quantifiers by their properties

• “every/all”, “all but (at most) n”: Left monotone decreasing, right monotone increasing.

• “a”, “some”, “at least n”, “infinitely many”: Left monotone increasing, right monotone increasing

• “no”, “at most n”, “finitely many”: Left monotone decreasing, right monotone decreasing

• “not every/all”: Left monotone increasing, right monotone decreasing.

Other quantifiers are not monotonic at all: E.g.

• “an even number” fails both, as does “exactly n” and “between m and n”

• Most is neither left increasing or decreasing, but right monotone increasing.

• “Many” is tricky. It may be left monotone increasing or not left monotone at all, but it is right monotone increasing. “Few” may or may not be left monotone decreasing, but it is right monotone decreasing. (Note “a few” fails even to be right monotone.)

• John is neither left increasing or decreasing, but right monotone increasing.

• Not John is neither left increasing or decreasing, but right monotone decreasing.

Note that Gamut (section 7.2) also refers to left monotonic as “persistent” or “anti-persistent” and right monotonic as “monotonic”.

It is interesting to see how the classification in this predicts linguistic constraints:

Constraint on Coordination: NP’s can be coordinated by conjunction and disjunction iff they have the same direction of right monotonicity.

Examples:

• A girl and many boys petted Rover.

• A girl and all boys petted Rover.

• Jane and all boys petted Rover.

• * Every girl and few boys petted Rover.
• * A girl and no boys petted Rover.
• * A girl and not all boys petted Rover.
• Few or no girls petted Rover.
• * All the boys and no girls petted Rover.
• * All the girls and few boys girls petted Rover.

The word “but” requires different direction of right monotonicity.

• Every girl, but few boys petted Rover.
• All the boys, but no girls petted the dog.

Unfortunately there are some exceptions:
• At least 2 girls and at most 3 boys petted Rover
• Jane and a few boys

Exercises:

1. Notice the following sentences involving “there”: 
   • There are some apples in my pocket.
   • *There is every apple in my pocket.

   Classify the quantifiers that do and do not work in these contexts.

2. The adverb “ever” is an example of a negative polarity item (NPI), because it seems to require a negative environment.
   • Jane hasn’t ever petted a dog.
   • *Jane has ever petted a dog.

Another example is “any”.
However, sometimes a “not” is not necessary to license “ever”. E.g.,
• Few people ever petted a dog.
• Everyone who ever petted the dog liked it.

Which properties of the determiners correlates with the distribution of ever?

**Dual quantifiers**

We can match the right monotonic quantifiers in pairs that have opposite directions:

• All and not all
• Some and no
• Many and few
We can define two different types of negations of quantifiers: internal and external:

**Definition 1** Let the binary relation $D$ represent a quantifier $Q$

1. The external negation of $Q$, written $\neg Q$, is defined by the relation $\neg D$ where $A \neg D B$ iff not $(A D B)$.

2. The external negation of $Q$, written $Q \neg$, is defined by the relation $D \neg$ where $A D \neg B$ iff not $(A D B)$, where $\overline{B}$ is the complement of $B$.

Let’s look at the internal and external negations of some quantifiers.

- The external negation of “all” is “not all”, while the internal negation is “no”.
- The external negation of “a” is “no”, while the internal negation is “not all”.
- The external negation of “at least n” is “fewer than n”, while the internal negation is “all but at most n”.

**Fact**: If $Q$ is right monotonic then both $\neg Q$ and $Q \neg$ are right monotonic, but in the opposite direction from $Q$.

Not surprisingly, double negations cancel if both are external or both are internal, but one of each is more interesting:

**Definition 2** The dual of $Q$, written $Q^*$ is defined to be $\neg Q \neg$ (the order of applying negations doesn’t matter). Thus $D^*$ is defined by $A D^* B$ iff not $A D \overline{B}$.

If $Q$ is right monotonic increasing (decreasing) then so is its dual, $Q^*$.

Some quantifiers are self-dual. For example, “at least half of all”

Some quantifiers allow using “not” to form an explicit external negation: “not every”, “not all”, not a single”, “not many”. Whereas others are not allowed: “not each”, “not some”, “not John”, “not the”. (To test these add all to “girl(s) walk(s)”).

Proposal (Barwise and Cooper): If a language has a syntactic construction whose semantic function is to negate a quantifier, then this construction will not be used with NP’s which are right monotonic decreasing or self-dual quantifier.